



#### INCLUDES

- ✓ Course framework
- ✓ Instructional section
- ✓ Sample exam questions

# AP<sup>®</sup> Physics C: Electricity and Magnetism

COURSE AND EXAM DESCRIPTION

Effective  
Fall 2024



# AP<sup>®</sup> Physics C: Electricity and Magnetism

---

## COURSE AND EXAM DESCRIPTION

Effective  
**Fall 2024**

**AP COURSE AND EXAM DESCRIPTIONS ARE UPDATED PERIODICALLY**

Please visit AP Central ([apcentral.collegeboard.org](https://apcentral.collegeboard.org)) to determine whether a more recent course and exam description is available.

# What AP® Stands For

Thousands of Advanced Placement teachers have contributed to the principles articulated here. These principles are not new; they are, rather, a reminder of how AP already works in classrooms nationwide. The following principles are designed to ensure that teachers' expertise is respected, required course content is understood, and that students are academically challenged and free to make up their own minds.

1. AP stands for clarity and transparency. Teachers and students deserve clear expectations. The Advanced Placement Program makes public its course frameworks and sample assessments. Confusion about what is permitted in the classroom disrupts teachers and students as they navigate demanding work.
2. AP is an unflinching encounter with evidence. AP courses enable students to develop as independent thinkers and to draw their own conclusions. Evidence and the scientific method are the starting place for conversations in AP courses.
3. AP opposes censorship. AP is animated by a deep respect for the intellectual freedom of teachers and students alike. If a school bans required topics from their AP courses, the AP Program removes the AP designation from that course and its inclusion in the AP Course Ledger provided to colleges and universities. For example, the concepts of evolution are at the heart of college biology, and a course that neglects such concepts does not pass muster as AP Biology.
4. AP opposes indoctrination. AP students are expected to analyze different perspectives from their own, and no points on an AP Exam are awarded for agreement with any specific viewpoint. AP students are not required to feel certain ways about themselves or the course content. AP courses instead develop students' abilities to assess the credibility of sources, draw conclusions, and make up their own minds.

As the AP English Literature course description states: "AP students are not expected or asked to subscribe to any one specific set of cultural or political values, but are expected to have the maturity to analyze perspectives different from their own and to question the meaning, purpose, or effect of such content within the literary work as a whole."

5. AP courses foster an open-minded approach to the histories and cultures of different peoples. The study of different nationalities, cultures, religions, races, and ethnicities is essential within a variety of academic disciplines. AP courses ground such studies in primary sources so that students can evaluate experiences and evidence for themselves.
6. Every AP student who engages with evidence is listened to and respected. Students are encouraged to evaluate arguments but not one another. AP classrooms respect diversity in backgrounds, experiences, and viewpoints. The perspectives and contributions of the full range of AP students are sought and considered. Respectful debate of ideas is cultivated and protected; personal attacks have no place in AP.
7. AP is a choice for parents and students. Parents and students freely choose to enroll in AP courses. Course descriptions are available online for parents and students to inform their choice. Parents do not define which college-level topics are suitable within AP courses; AP course and exam materials are crafted by committees of professors and other expert educators in each field. AP courses and exams are then further validated by the American Council on Education and studies that confirm the use of AP scores for college credits by thousands of colleges and universities nationwide.

The AP Program encourages educators to review these principles with parents and students so they know what to expect in an AP course. Advanced Placement is always a choice, and it should be an informed one. AP teachers should be given the confidence and clarity that once parents have enrolled their child in an AP course, they have agreed to a classroom experience that embodies these principles.

# Contents

---

- v **Acknowledgments**
- 1 **About AP**
- 4 **AP Resources and Supports**
- 5 **Instructional Model**
- 6 **About the AP Physics C: Electricity and Magnetism Course**
- 6 College Course Equivalent
- 6 Prerequisites
- 6 Lab Requirement

---

## **COURSE FRAMEWORK**

- 9 **Introduction**
- 10 **Course Framework Components**
- 11 Science Practices
- 13 Course Content
- 15 **Course at a Glance**
- 19 **Unit Guides**
- 20 Using the Unit Guides
- 23 **UNIT 8:** Electric Charges, Fields, and Gauss's Law
- 39 **UNIT 9:** Electric Potential
- 49 **UNIT 10:** Conductors and Capacitors
- 61 **UNIT 11:** Electric Circuits
- 83 **UNIT 12:** Magnetic Fields and Electromagnetism
- 97 **UNIT 13:** Electromagnetic Induction

---

## **LABORATORY INVESTIGATIONS**

- 115 **Lab Experiments**
- 116 **How to Set Up a Lab Program**

---

## **INSTRUCTIONAL APPROACHES**

- 125 **Selecting and Using Course Materials**
- 126 **Guided Inquiry in AP Physics C: Electricity and Magnetism**
- 129 **Instructional Strategies**
- 134 **Developing the Science Practices**

---

## **EXAM INFORMATION**

- 145 **Exam Overview**
- 147 **How Student Learning Is Assessed on the AP Exam**
- 150 **Task Verbs Used in Free-Response Questions**
- 151 **Sample Exam Questions**

---

### **SCORING GUIDELINES**

- 169 **Question 1: Mathematical Routines**
- 171 **Question 2: Translation Between Representations**
- 173 **Question 3: Experimental Design and Analysis**
- 175 **Question 4: Qualitative/Quantitative Translation**

---

### **APPENDIX**

- 183 **Table of Information: Equations**

# Acknowledgments

---

College Board would like to acknowledge the following committee members, consultants, and reviewers for their assistance with and commitment to the development of this course. All individuals and their affiliations were current at the time of contribution.

**Hamza Balci**, *Kent State University, Kent, OH*  
**Suzanne Brahmia**, *University of Washington, Seattle, WA*  
**Eric Burkholder**, *Stanford University, Stanford, CA*  
**Gerald Cleaver**, *Baylor University, Waco, TX*  
**Robert Davis**, *Brigham Young University, Provo, UT*  
**Veronica Ellingson**, *Hudson Senior High School, Hudson, WI*  
**Karin Foss**, *Wayzata High School, Plymouth, MN*  
**Herbert Fotso**, *The State University of New York at Buffalo, Buffalo, NY*  
**Kristin Glover**, *Prairie Ridge High School, Crystal Lake, Illinois*  
**Kathy Harper**, *Case Western Reserve University, Cleveland, OH*  
**Mark Hossler**, *Landmark Christian School, Fairburn, GA*  
**Stefan Jeglinski**, *University of North Carolina, Chapel Hill, NC*  
**Kathy Koenig**, *University of Cincinnati, Cincinnati, OH*  
**Kristine Lang**, *Colorado College, Colorado Springs, CO*  
**Robynne Lock**, *Texas A&M University, Commerce, Commerce, TX*  
**Sam Lofland**, *Rowan University, Glassboro, New Jersey*  
**Joe Mancino**, *Glastonbury High School, Glastonbury, CT*  
**Ricardo Markland**, *Miami Coral Park Senior High School, Miami, FL*  
**Dee Dee Messer**, *William Mason High School, Mason, OH*  
**Matthew Milligan**, *Farragut High School, Knoxville, TN*  
**Holley Mosley**, *Liberty High School, Frisco, TX*  
**Heather Rave**, *St. Thomas Aquinas College, Orangeburg, NY*  
**Matt Sckalor**, *Great Neck South High School, Great Neck, NY*  
**Peter Sheldon**, *Randolph College, Lynchburg, VA*  
**Gay Stewart**, *West Virginia University, Morgantown, WV*  
**Shelly Strand**, *West Fargo High School, West Fargo, ND*  
**Dawn Striker**, *Longmeadow High School, Longmeadow, MA*  
**Oather Strawderman**, *Lawrence Free State High School, Lawrence, KS*  
**Yau-Jong Twu**, *Eleanor Roosevelt High School, Greenbelt, MD*  
**Brian Utter**, *University of California Merced, Merced, CA*  
**Matthew Vonk**, *University of Wisconsin-River Falls, River Falls, WI*

## College Board Staff

**Amy Johnson**, *Director, AP Physics Curriculum and Assessment*

**Jesse Miner**, *Director, AP Physics Curriculum and Assessment*

**John Pinizzotto**, *Director, AP Physics Curriculum and Assessment*

**Shu-Kang Chen**, *Executive Director, AP STEM Curriculum and Assessment*

**Laura Keegan**, *Director, Product Development and Editorial, AP Classroom Instruction*

**Claire Lorenz**, *Senior Director, AP Instruction*

**Serena Magrogan**, *Senior Director, AP Science Department Head*

**Daniel McDonough**, *Senior Director, AP Content and Assessment Publications*

**Allison Thurber**, *Vice President, AP Curriculum and Assessment*

**SPECIAL THANKS** *Brian Brown, Anders Gardestig, Ann Marie Zolanz, Angela Jensvold, and Mark Widmer*

# About AP

---

The Advanced Placement® Program (AP®) enables willing and academically prepared students to pursue college-level studies—with the opportunity to earn college credit, advanced placement, or both—while still in high school. Through AP courses in 40 subjects, each culminating in a challenging exam, students learn to think critically, construct solid arguments, and see many sides of an issue—skills that prepare them for college and beyond. Taking AP courses demonstrates to college admission officers that students have sought the most challenging curriculum available to them, and research indicates that students who score a 3 or higher on an AP Exam typically experience greater academic success in college and are more likely to earn a college degree than non-AP students. Each AP teacher’s syllabus is evaluated and approved by faculty from some of the nation’s leading colleges and universities, and AP Exams are developed and scored by college faculty and experienced AP teachers. Most four-year colleges and universities in the United States grant credit, advanced placement, or both on the basis of successful AP Exam scores—more than 3,300 institutions worldwide annually receive AP scores.

## AP Course Development

In an ongoing effort to maintain alignment with best practices in college-level learning, AP courses and exams emphasize challenging, research-based curricula aligned with higher education expectations.

Individual teachers are responsible for designing their own curriculum for AP courses, selecting appropriate college-level readings, assignments, and resources. This course and exam description presents the content and skills that are the focus of the corresponding college course and that appear on the AP Exam. It also organizes the content and skills into a series of units that represent a sequence found in widely adopted college textbooks and that many AP teachers have told us they follow in order to focus their instruction. The intention of this publication is to respect teachers’ time and expertise by providing a roadmap that they can modify and adapt to their local priorities and preferences. Moreover, by organizing the AP course content and skills into units, the AP Program is able to provide teachers and students with free formative assessments—Progress Checks—that teachers

can assign throughout the year to measure student progress as they acquire content knowledge and develop skills.

## Enrolling Students: Equity and Access

The AP Program strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial, and socioeconomic groups that have been traditionally underserved. The AP Program also believes that all students should have access to academically challenging coursework before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

## Offering AP Courses: The AP Course Audit

The AP Program unequivocally supports the principle that each school implements its own curriculum that will enable students to develop the content understandings and skills described in the course framework.

While the unit sequence represented in this publication is optional, the AP Program does have a short list of curricular and resource requirements that must be fulfilled before a school can label a course “Advanced Placement” or “AP.” Schools wishing to offer AP courses must participate in the AP Course Audit, a process through which AP teachers’ course materials are reviewed by college faculty. The AP Course Audit was created to provide teachers and administrators with clear guidelines on curricular and resource requirements for AP courses and to help colleges and universities validate courses marked “AP” on students’ transcripts. This process ensures that AP teachers’ courses meet or exceed the curricular and resource expectations that college and secondary school faculty have established for college-level courses.



The AP Course Audit form is submitted by the AP teacher and the school principal (or designated administrator) to confirm awareness and understanding of the curricular and resource requirements. A syllabus or course outline, detailing how course requirements are met, is submitted by the AP teacher for review by college faculty.

Please visit [collegeboard.org/apcourseaudit](https://collegeboard.org/apcourseaudit) for more information to support the preparation and submission of materials for the AP Course Audit.

## How the AP Program Is Developed

The scope of content for an AP course and exam is derived from an analysis of hundreds of syllabi and course offerings of colleges and universities. Using this research and data, a committee of college faculty and expert AP teachers work within the scope of the corresponding college course to articulate what students should know and be able to do upon the completion of the AP course. The resulting course framework is the heart of this course and exam description and serves as a blueprint of the content and skills that can appear on an AP Exam.

The AP Test Development Committees are responsible for developing each AP Exam, ensuring the exam questions are aligned to the course framework. The AP Exam development process is a multiyear endeavor; all AP Exams undergo extensive review, revision, piloting, and analysis to ensure that questions are accurate, fair, and valid, and that there is an appropriate spread of difficulty across the questions.

Committee members are selected to represent a variety of perspectives and institutions (public and private, small and large schools and colleges), and a range of gender, racial/ethnic, and regional groups. A list of each subject’s current AP Test Development Committee members is available on [apcentral.collegeboard.org](https://apcentral.collegeboard.org).

Throughout AP course and exam development, College Board gathers feedback from various stakeholders in both secondary schools and higher education institutions. This feedback is carefully considered to ensure that AP courses and exams are able to provide students with a college-level learning experience and the opportunity to demonstrate their qualifications for advanced placement or college credit.

## How AP Exams Are Scored

The exam scoring process, like the course and exam development process, relies on the expertise of both AP teachers and college faculty. While multiple-choice questions are scored by machine, the free-response questions and through-course performance assessments, as applicable, are scored by thousands of college faculty and expert AP

teachers. Most are scored at the annual AP Reading, while a small portion is scored online. All AP Readers are thoroughly trained, and their work is monitored throughout the Reading for fairness and consistency. In each subject, a highly respected college faculty member serves as Chief Faculty Consultant and, with the help of AP Readers in leadership positions, maintains the accuracy of the scoring standards. Scores on the free-response questions and performance assessments are weighted and combined with the results of the computer-scored multiple-choice questions, and this raw score is converted into a composite AP score on a 1–5 scale.

AP Exams are **not** norm-referenced or graded on a curve. Instead, they are criterion-referenced, which means that every student who meets the criteria for an AP score of 2, 3, 4, or 5 will receive that score, no matter how many students that is. The criteria for the number of points students must earn on the AP Exam to receive scores of 3, 4, or 5—the scores that research consistently validates for credit and placement purposes—include:

- The number of points successful college students earn when their professors administer AP Exam questions to them.
- Performance that researchers have found to be predictive of an AP student succeeding when placed into a subsequent higher-level college course.
- The number of points college faculty indicate, after reviewing each AP question, that they expect is necessary to achieve each AP grade level.

## Using and Interpreting AP Scores

The extensive work done by college faculty and AP teachers in the development of the course and exam and throughout the scoring process ensures that AP Exam scores accurately represent students’ achievement in the equivalent college course. Frequent and regular research studies establish the validity of AP scores as follows:

AP Score	Credit Recommendation	College Grade Equivalent
5	Extremely well qualified	A
4	Well qualified	A–, B+, B
3	Qualified	B–, C+, C
2	Possibly qualified	n/a
1	No recommendation	n/a

While colleges and universities are responsible for setting their own credit and placement policies, most private colleges and universities award credit and/or advanced placement for AP scores of 3 or higher. Additionally, most states in the U.S. have adopted statewide credit policies that ensure college credit for scores of 3 or higher at public colleges and universities. To confirm a specific college's AP credit/placement policy, a search engine is available at [apstudent.collegeboard.org/creditandplacement/search-credit-policies](https://apstudent.collegeboard.org/creditandplacement/search-credit-policies).

### **BECOMING AN AP READER**

Each June, thousands of AP teachers and college faculty members from around the world gather for seven days in multiple locations to evaluate and score the free-response sections of the AP Exams. Ninety-eight percent of surveyed educators who took part in the AP Reading say it was a positive experience.

There are many reasons to consider becoming an AP Reader, including opportunities to:

- **Bring positive changes to the classroom:** Surveys show that the vast majority of returning AP Readers—both high school and college

educators—make improvements to the way they teach or score because of their experience at the AP Reading.

- **Gain in-depth understanding of AP Exam and AP scoring standards:** AP Readers gain exposure to the quality and depth of the responses from the entire pool of AP Exam takers, and thus are better able to assess their students' work in the classroom.
- **Receive compensation:** AP Readers are compensated for their work during the Reading. Expenses, lodging, and meals are covered for Readers who travel.
- **Score from home:** AP Readers have online distributed scoring opportunities for certain subjects. Check [collegeboard.org/apreading](https://collegeboard.org/apreading) for details.
- **Earn Continuing Education Units (CEUs):** AP Readers earn professional development hours and CEUs that can be applied to PD requirements by states, districts, and schools.

### **How to Apply**

Visit [collegeboard.org/apreading](https://collegeboard.org/apreading) for eligibility requirements and to start the application process.

# AP Resources and Supports

---

By completing a simple class selection process at the start of the school year, teachers and students receive access to a robust set of classroom resources.

## AP Classroom

**AP Classroom** is a dedicated online platform designed to support teachers and students throughout their AP experience. The platform provides a variety of powerful resources and tools to provide yearlong support to teachers and students, offering opportunities to give and get meaningful feedback on student progress.



### UNIT GUIDES

Appearing in this publication and on AP Classroom, these planning guides outline all required course content and skills, organized into commonly taught units. Each Unit Guide suggests a sequence and pacing of content, scaffolds skill instruction across units, organizes content into topics, and provides tips on taking the AP Exam.



### PROGRESS CHECKS

Formative AP questions for every unit provide feedback to students on the areas where they need to focus. Available online, Progress Checks measure knowledge and skills through multiple-choice questions with rationales to explain correct and incorrect answers, and free-response questions with scoring information. Because the Progress Checks are formative, the results of these assessments cannot be used to evaluate teacher effectiveness or assign letter grades to students, and any such misuses are grounds for losing school authorization to offer AP courses.\*



### REPORTS

The reports section provides teachers with a one-stop shop for student results on all assignment types, including Progress Checks. Teachers can view class trends and see where students struggle with content and skills that will be assessed on the AP Exam. Students can view their own progress over time to improve their performance before the AP Exam.



### QUESTION BANK

The Question Bank is a searchable library of all AP questions that teachers use to build custom practice for their students. Teachers can create and assign assessments with formative topic questions or questions from practice or released AP Exams.

#### Class Section Setup and Enrollment

- Teachers and students sign in to or create their College Board accounts.
- Teachers confirm that they have added the course they teach to their AP Course Audit account and have had it approved by their school's administrator.
- Teachers or AP coordinators, depending on who the school has decided is responsible, set up class sections so students can access AP resources and have exams ordered on their behalf.
- Students join class sections with a join code provided by their teacher or AP coordinator.
- Students will be asked for additional information upon joining their first class section.

---

\*To report misuses, please call, 877-274-6474 (International: 212-632-1781).

# Instructional Model

---

Integrating AP resources throughout the course can help students develop skills and conceptual understandings. The instructional model outlined below shows possible ways to incorporate AP resources into the classroom.



## Plan

Teachers may consider the following approaches as they plan their instruction before teaching each unit.

- Review the overview at the start of each **Unit Guide** to identify essential questions, conceptual understandings, and skills for each unit.
- Use the **Unit at a Glance** table to identify related topics that build toward a common understanding, and then plan appropriate pacing for students.
- Identify useful strategies in the **Instructional Approaches** section to help teach the concepts and skills.



## Teach

When teaching, supporting resources could be used to build students' conceptual understanding and their mastery of skills.

- Use the topic pages in the **Unit Guides** to identify the required content.
- Integrate the content with a skill, considering any appropriate scaffolding.
- Employ any of the instructional strategies previously identified.
- Use the available resources, including **AP Daily**, on the topic pages to bring a variety of assets into the classroom.



## Assess

Teachers can measure student understanding of the content and skills covered in the unit and provide actionable feedback to students.

- As you teach each topic, use **AP Classroom** to assign student **Topic Questions** as a way to continuously check student understanding and provide just in time feedback
- At the end of each unit, use **AP Classroom** to assign students **Progress Checks**, as homework or an in-class task.
- Provide question-level feedback to students through answer rationales; provide unit- and skill-level formative feedback using **Reports**.
- Create additional practice opportunities using the **Question Bank** and assign them through **AP Classroom**.

# About the AP Physics C: Electricity and Magnetism Course

---

AP Physics C: Electricity and Magnetism is a calculus-based introductory college-level physics course. Students cultivate their understanding of physics by developing models of physical phenomena through inquiry-based investigations.

Students build their understanding of physical models as they explore and solve problems in these topics:

- Electric Charges, Fields, and Gauss's Law
- Electric Potential
- Conductors and Capacitors
- Electric Circuits
- Magnetic Fields and Electromagnetism
- Electromagnetic Induction

## College Course Equivalent

AP Physics C: Electricity and Magnetism is equivalent to the second course in an introductory college course sequence in calculus-based physics.

## Prerequisites

Students should have taken or be concurrently taking calculus. Students should have taken AP Physics C: Mechanics, AP Physics 1 or another mechanics-based physics course prior to taking AP Physics C: Electricity and Magnetism.

## Laboratory Requirement

This course requires that 25 percent of instructional time be spent in hands-on laboratory work, with an emphasis on inquiry-based investigations that provide students with opportunities to demonstrate the foundational physics principles and apply the science practices.

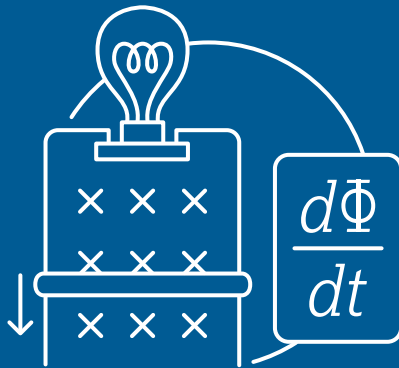
Inquiry-based laboratory experiences support the AP Physics C: Electricity and Magnetism course and AP Course Audit curricular requirements by providing opportunities for students to engage in the science practices as they design plans for experiments, make predictions, collect and analyze data, apply mathematical routines, develop explanations, and communicate about their work.

Colleges may require students to present their laboratory materials from AP science courses before granting college credit for laboratory work, so students should be encouraged to retain their laboratory notebooks, reports, and other materials.

**AP PHYSICS C: ELECTRICITY AND MAGNETISM**

---

# Course Framework



# Introduction

---

*The AP Physics C: Electricity and Magnetism course outlined in this framework reflects a commitment to what physics teachers, professors, and researchers have agreed is the main goal of a college level physics course: to help students develop a deep understanding of the foundational principles that shape classical mechanics. By confronting complex physical situations or scenarios, the course is designed to enable students to develop the ability to reason about physical phenomena using important science practices, such as explaining relationships, applying and justifying the use of mathematical routines, designing experiments, analyzing data, and making connections across multiple topics within the course.*

To foster this deeper level of learning, the AP Physics C: Electricity and Magnetism course defines concepts, skills, and understandings required by representative colleges and/or universities for granting college credit and placement. Students will practice reasoning skills used by physicists by discussing and debating, with peers, the physical phenomena investigated in class, as well as by designing and conducting inquiry-based laboratory investigations to solve problems through first hand observations, data collection, analysis, and interpretation.

This document is not a complete curriculum. Teachers create their own local curriculum by selecting, for each concept, content that enables students to explore the course learning objectives and meets state or local requirements. This results in a course that prepares students for college credit and/or placement.



# Course Framework Components

---

## Overview

This course framework provides a clear and detailed description of the course requirements necessary for student success. The framework specifies what students must know, be able to do, and understand to qualify for college credit and/or placement.

### The course framework includes two essential components:

#### 1 SCIENCE PRACTICES

The science practices are central to the study and practice of physics. Students should develop and apply the described practices on a regular basis over the span of the course.

#### 2 COURSE CONTENT

The course content is organized into commonly taught units of study that provide a suggested sequence for the course, and detail required content and conceptual understandings that colleges and universities typically expect students to be proficient in, in order to qualify for college credit and/or placement.

# 1

## AP PHYSICS C: ELECTRICITY AND MAGNETISM

# Science Practices

---

The table that follows presents the science practices that students should develop during the AP Physics C: Electricity and Magnetism course. These practices, and their related skills, form the basis of many tasks on the AP Physics C: Electricity and Magnetism Exam.

The unit guides that follow embed and spiral these science practices throughout the course, providing teachers with one way to integrate the skills into the course content with sufficient repetition to prepare students to apply those skills when taking the AP Physics C: Electricity and Magnetism Exam.

More detailed information about teaching the science practices can be found in the [Instructional Approaches](#) section of this publication.



# Science Practices

## Practice 1

### *Creating Representations* **1**

Create representations that depict physical phenomena.

#### SKILLS

**1.A** Create diagrams, tables, charts, or schematics to represent physical situations.

**1.B** Create quantitative graphs with appropriate scales and units, including plotting data.

**1.C** Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

## Practice 2

### *Mathematical Routines* **2**

Conduct analyses to derive, calculate, estimate, or predict.

**2.A** Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

**2.B** Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

**2.C** Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

**2.D** Predict new values or factors of change of physical quantities using functional dependence between variables.

## Practice 3

### *Scientific Questioning and Argumentation* **3**

Describe experimental procedures, analyze data, and support claims.

**3.A** Create experimental procedures that are appropriate for a given scientific question.

**3.B** Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

**3.C** Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## 2

# AP PHYSICS C: ELECTRICITY AND MAGNETISM

# Course Content

---

The AP Physics C: Electricity and Magnetism course framework provides a clear detailed description of the course requirements for student success. The framework specifies what students must know, be able to do, and understand with a focus on ideals that encompass core principles, theories, and processes of physics. This framework also encourages instruction that prepares students to make connections across domains through a broader way of thinking about the physical world.

## UNITS

The course content is organized into commonly taught units. The units have been arranged in a logical sequence frequently found in many college courses and textbooks.

The six units in AP Physics C: Electricity and Magnetism and their relevant weightings on the multiple-choice section of the AP Exam are listed on the next page.

Pacing recommendations on the Course at a Glance page provide suggestions for how teachers can cover both the required course content and the Progress Checks. The number of suggested class periods is based on a schedule in which the class meets five days a week for 45 minutes each day or for 90 minutes a day for a single semester. While these recommendations have been made to aid in planning, teachers are free to adjust the pacing based on the needs of their students, alternate schedules (e.g., block scheduling), or their school's academic calendar.

## Exam Weighting for the Multiple-Choice Section of the AP Physics C: Electricity and Magnetism Exam

Units of Instruction	Exam Weighting
<b>Unit 8:</b> <i>Electric Charges, Fields, and Gauss’s Law</i>	<b>15–25%</b>
<b>Unit 9:</b> <i>Electric Potential</i>	<b>10–20%</b>
<b>Unit 10:</b> <i>Conductors and Capacitors</i>	<b>10–15%</b>
<b>Unit 11:</b> <i>Electric Circuits</i>	<b>15–25%</b>
<b>Unit 12:</b> <i>Magnetic Fields and Electromagnetism</i>	<b>10–20%</b>
<b>Unit 13:</b> <i>Electromagnetic Induction</i>	<b>10–20%</b>

## TOPICS

Each unit is divided into teachable segments called topics. Visit the topic pages (starting on page 23) to see all the required content for each topic.

### Learning Objectives and Science Practices

In the AP Physics C: Electricity and Magnetism course and exam, every exam question will be aligned to a learning objective and a skill. The learning objectives represent the content domain, while the skill represents the science practice required to successfully complete the task. The three categories of science practices are described as discrete practices; but they are in fact interrelated. For example, scientific questions and predictions are associated with underlying mathematical relationships, and those relationships are used to create diagrams and graphs. The ordering of the science practices is not meant to describe any hierarchy of importance or difficulty.

The three science practices, and their associated skills, will be applied to all learning objectives in the course framework. The task verb “describe,” which is used in nearly all learning objectives, encompasses the range of possible graphical, mathematical, or verbal skill applications. Within these multiple representations, students should be able to “describe” a physical concept graphically, mathematically, and verbally.

For example, for a given learning objective, teachers are encouraged to ask the following questions about a physical phenomenon:

- How would students create or interpret graphs or other visual representations?
- What quantitative problems could students solve?
- What experiment could a student design, or what data would students analyze?
- How could the concepts be described verbally?
- How could course content be used as evidence to justify or support a claim about the behavior of a system, physical phenomena, or outcome of an experiment?

### Required Equations

Not all equations in this course framework appear on the equation sheet provided to students while taking the AP Physics C: Electricity and Magnetism Exam. Many of the equations in this document are provided for reference and guidance, or to demonstrate the final results of derivations expected of students on the exam. These equations are denoted as “Derived Equations.” Additionally, variables used within this course framework follow the definitions given on the equation sheet. For a complete list of the equations available to students on the AP Physics C: Electricity and Magnetism Exam, please see the AP Physics C: Electricity and Magnetism Table of Information: Equations in the [Appendix](#).

# Course at a Glance

## Plan

The Course at a Glance provides a useful visual organization for the AP Physics C: Electricity and Magnetism course components, including:

- Sequence of units, along with approximate weighting and suggested pacing. Please note, pacing is based on 45-minute class periods, meeting five days each week for a full academic year and for 90 minute periods meeting daily for a single semester.
- Progression of topics within each unit.
- Spiraling of the science practices across units.

## Teach

### PRACTICES

Science Practices spiral throughout the course

- 1 Creating Representations
- 2 Mathematical Routines
- 3 Scientific Questioning and Argumentation

## Required Course Content

Each topic contains required Learning Objectives and Essential Knowledge Statements that form the basis of the assessment on the AP Exam.

## Assess

Assign the Progress Checks—either as homework or in class—for each unit. Each Progress Check contains formative multiple-choice and free-response questions. The feedback from these checks shows students the areas where they need to focus.

**UNIT 8** Electric Charges, Fields, and Gauss's Law  
~12/~24 Class Periods 15–25% AP Exam Weighting

- 1 2 3 8.1 Electric Charge and Electric Force
- 1 2 3 8.2 Conservation of Electric Charge and the Process of Charging
- 1 2 3 8.3 Electric Fields
- 1 2 3 8.4 Electric Fields of Charge Distributions
- 1 2 3 8.5 Electric Flux
- 1 2 3 8.6 Gauss's Law

**UNIT 9** Electric Potential  
~10/~20 Class Periods 10–20% AP Exam Weighting

- 1 2 3 9.1 Electric Potential Energy
- 1 2 3 9.2 Electric Potential
- 1 2 3 9.3 Conservation of Electric Energy

### Progress Check 8

Multiple-choice: ~18 questions

Free-response: 4 questions

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

### Progress Check 9

Multiple-choice: ~18 questions

Free-response: 4 questions

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# UNIT 10

## Conductors and Capacitors

~8/~16

Class  
Periods

10–15%

AP Exam  
Weighting

1  
2  
3

### 10.1 Electrostatics with Conductors

1  
2  
3

### 10.2 Redistribution of Charge between Conductors

1  
2  
3

### 10.3 Capacitors

1  
2  
3

### 10.4 Dielectrics

# UNIT 11

## Electric Circuits

~12/~24

Class  
Periods

15–25%

AP Exam  
Weighting

1  
2  
3

### 11.1 Electric Current

1  
2  
3

### 11.2 Simple Circuits

1  
2  
3

### 11.3 Resistance, Resistivity, and Ohm's Law

1  
2  
3

### 11.4 Electric Power

1  
2  
3

### 11.5 Compound Direct Current Circuits

1  
2  
3

### 11.6 Kirchhoff's Loop Rule

1  
2  
3

### 11.7 Kirchhoff's Junction Rule

1  
2  
3

### 11.8 Resistor Capacitor (RC) Circuits

# UNIT 12

## Magnetic Fields and Electromagnetism

~10/~20

Class  
Periods

10–20%

AP Exam  
Weighting

1  
2  
3

### 12.1 Magnetic Fields

1  
2  
3

### 12.2 Magnetism and Moving Charges

1  
2  
3

### 12.3 Magnetic Fields of Current-Carrying Wires and the Biot-Savart Law

1  
2  
3

### 12.4 Ampère's Law

### Progress Check 10

Multiple-choice: ~18 questions

Free-response: 4 questions

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

### Progress Check 11

Multiple-choice: ~24 questions

Free-response: 4 questions

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

### Progress Check 12

Multiple-choice: ~18 questions

Free-response: 4 questions

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# UNIT 13

## Electromagnetic Induction

**~10/~20** Class Periods | **10–20%** AP Exam Weighting

1  
2  
3

### 13.1 Magnetic Flux

1  
2  
3

### 13.2 Electromagnetic Induction

1  
2  
3

### 13.3 Induced Currents and Magnetic Forces

1  
2  
3

### 13.4 Inductance

1  
2  
3

### 13.5 Circuits with Resistors and Inductors (LR Circuits)

1  
2  
3

### 13.6 Circuits with Capacitors and Inductors (LC Circuits)

#### Progress Check 13

**Multiple-choice: ~18 questions**

**Free-response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation



THIS PAGE IS INTENTIONALLY LEFT BLANK.

## AP PHYSICS C: ELECTRICITY AND MAGNETISM

# Unit Guides

---

### Introduction

Designed with input from the community of AP Physics C: Electricity and Magnetism educators, the Unit Guides offer teachers helpful guidance in building students' skills and content knowledge. The suggested sequence was identified through a thorough analysis of the syllabi of highly effective AP teachers and the organization of typical college textbooks.

This unit structure respects new AP teachers' time by providing one possible sequence they can adopt or modify rather than having to build from scratch. An additional benefit is that these units enable the AP Program to provide interested teachers with formative assessments—the Progress Checks—that they can assign their students at the end of each unit to gauge progress toward success on the AP Exam. However, experienced AP teachers who are pleased with their current course organization and exam results should feel no pressure to adopt these units, which comprise an optional sequence for this course.

# Using the Unit Guides

**UNIT 8** 15–25% AP EXAM WEIGHTING ~12/~24 CLASS PERIODS

## Electric Charges, Fields, and Gauss’s Law

**Developing Understanding**

In Unit 8, students will begin the study of electric force, which is exerted on all objects with a property called charge. The electric force, in contrast to gravitational force, is one of attraction or repulsion and, therefore, leads to different effects on objects. This knowledge will help students understand the role electrostatics plays in common devices such as photocopiers, defibrillators, and printers, as well as television, radio, and radar industries. In the units that follow, students will apply their knowledge of electric charges and force to electric circuits, and how the motion of electric charges creates magnetic fields.

**Building the Science Practices**

Unit 8 provides multiple opportunities for students to create and use visual representations (1.A and 1.C) to demonstrate an understanding of the relationships between the variables that describe the motion of objects or systems. Unit 8 will also reinforce the importance of demonstrating consistency between different graphical representations of the same physical situation. Being able to identify, create, and use graphs that represent the same physical situation demonstrates a deeper understanding of concepts than simply creating or using one representation. As students incorporate these skills in Unit 8, their abilities to identify consistencies between graphs as well as to create graphs that are consistent with each other will improve with continued practice. Lastly, throughout the unit, students will be challenged to identify which fundamental law, definition, and/or mathematical relationship will apply to a given scenario. Selecting the appropriate relationship and subsequent solution technique are critical problem-solving skills that should be given space to be developed as a first step towards deriving equations (2.A) to represent physical scenarios.

**Preparing for the AP Exam**

The first question on the AP Physics C: Electricity and Magnetism Exam—the Mathematical Routines (MR) question—focuses on assessing students’ ability to create and use mathematical models and representations. Students will be required to derive an expression for a physical quantity which may culminate in a numerical calculation. Students will expand on these mathematical skills by either creating a representation of the scenario at hand, or making and justifying claims about that same scenario. While Unit 8 offers content perfect for practicing the MR question, the MR question on the AP Physics C: Electricity and Magnetism Exam can pull content from any of the six units of the course.

**ESSENTIAL QUESTIONS**

- Why does hair stand up after brushing it with a plastic comb?
- How does a charged rubber rod bend a stream of water?
- How might it be possible to get a balloon to stick to the wall?
- Why don’t cell phones work in concrete buildings?
- Why can a bird land safely on a high voltage wire?

AP Physics C: Electricity and Magnetism Course and Exam Description Course Framework V.1 | 25

## UNIT OPENERS

**Developing Understanding** provides an overview that contextualizes and situates the key content of the unit within the scope of the course.

The **essential questions** are thought-provoking questions that motivate students and inspire inquiry.

**Building the Science Practices** described specific skills within the practices that are appropriate to focus on in that unit. Certain practices have been noted to indicate areas of emphasis for that unit.

**Preparing for the AP Exam** provides helpful tips and common student misunderstandings identified from prior exam data.

**UNIT 8** Electric Charges, Fields, and Gauss’s Law

## UNIT AT A GLANCE

Topics	Suggested Skills
<b>8.1 Electric Charge and Electric Force</b>	<ul style="list-style-type: none"> <li>1.A Create diagrams, tables, charts, or schematics to represent physical situations.</li> <li>1.B Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</li> <li>1.C Predict new values or factors of change of physical quantities using functional dependence between variables.</li> <li>2.A Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li> </ul>
<b>8.2 Conservation of Electric Charge and the Process of Charging</b>	<ul style="list-style-type: none"> <li>1.A Create diagrams, tables, charts, or schematics to represent physical situations.</li> <li>1.B Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li> <li>1.C Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li> <li>2.A Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li> </ul>
<b>8.3 Electric Fields</b>	<ul style="list-style-type: none"> <li>1.A Create quantitative graphs with appropriate scales and units, including plotting data.</li> <li>1.B Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li> <li>1.C Predict new values or factors of change of physical quantities using functional dependence between variables.</li> <li>2.A Create experimental procedures that are appropriate for a given scientific question.</li> <li>2.B Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li> </ul>
<b>8.4 Electric Fields of Charge Distributions</b>	<ul style="list-style-type: none"> <li>1.A Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</li> <li>1.B Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li> <li>1.C Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li> <li>2.A Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li> </ul>

*continued on next page*

AP Physics C: Electricity and Magnetism Course and Exam Description Course Framework V.1 | 26

The **Unit at a Glance** table shows the topics, related enduring understandings, and suggested skills.

The **suggested skills** for each topic show possible ways to link content in that topic to specific AP Physics skills. The individual skills have been thoughtfully chosen in a way that scaffolds skills throughout the course. The questions on the Progress Checks are based on this pairing. However, AP Exam questions can pair the content with any of the skills.

# Using the Unit Guides

**UNIT 8**      **Electric Charges, Fields, and Gauss's Law**

### SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	8.1	<b>Desktop Experiment Tasks</b> Give two pith balls some amount of charge (assumed to be equal charges) and hang the balls from string near enough to each other that they exert a repulsive force on each other. Have students measure the angle their strings make and use this information to determine the charge on the pith balls. Also have them determine what fraction of the pith ball's electrons were lost/gained assuming one electron for every $3.3 \times 10^{-27}$ kg of mass.
2	8.1	<b>Qualitative Reasoning</b> Have students consider a situation where two metal spheres (one heavy, one light) have unequal magnitudes, opposite charges, and are set at rest near each other in space. Then, have students draw acceleration versus time and velocity versus time graphs for the times when the light sphere attracts, collides elastically with, and then repels from the heavier sphere.
3	8.2	<b>Desktop Experiment Tasks</b> Have students place sticky tape, sticky side down, on the tops of their desks. Then, have students place a second piece of tape on top of the first and then a third piece on top of the second. Have students quickly remove pieces 2 and 3 simultaneously from the desk. Then, quickly separate pieces 2 and 3 from each other. Have students predict whether the pieces of tape will be charged and if they will have different or similar charges.
4	8.3	<b>Desktop Experiment Tasks</b> Have students play the PHET simulation "Electric Field Hockey." Have a competition to see which group of students can score a goal with the least number of charges guiding the test particle.
5	8.6	<b>Create a Plan</b> Have students research the electric field strength and direction at ground level on Earth. Next, have them use Gauss's Law to determine the net charge on Earth.

AP Physics C: Electricity and Magnetism Course and Exam Description      Course Framework V.1 | 28

The **Sample Instructional Activities** page includes optional activities that can help teachers tie together the content and skill for a particular topic.

**Electric Charges, Fields, and Gauss's Law**      **UNIT 8**

## TOPIC 8.4 Electric Fields of Charge Distributions

**Required Course Content**

**LEARNING OBJECTIVE**

**8.A.A.** Describe the electric field resulting from a given charge distribution.

**ESSENTIAL KNOWLEDGE**

**8.A.A.1** Expressions for the electric field of specified charge distributions can be found using integration and the principle of superposition.  
Relevant equation:  
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

**8.A.A.2** Symmetry considerations of certain charge distributions can simplify analysis of the electric field resulting from those charge distributions.

**BOUNDARY STATEMENT**

*AP Physics C: Electricity & Magnetism only expects students to use calculus to find the electric field resulting from the following charge distributions and locations: an infinitely long, uniformly charged wire or cylinder at a distance from its central axis, a thin ring of charge at a location along the axis of the ring, a semicircular arc or part of a semicircular arc at its center, and a finite wire or line charge at a point collinear with the line charge or at a location along its perpendicular bisector.*

**SUGGESTED SKILLS**

**1.C.1** Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

**2.E.1** Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

**3.C.1** Compare physical quantities between two or more scenarios of all different times and locations in a single scenario.

**4.B.1** Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

AP Physics C: Electricity and Magnetism Course and Exam Description      Course Framework V.1 | 35

## TOPIC PAGES

The **suggested skills** offer possible skills to pair with the topic.

**Learning objectives** define what a student needs to be able to do with content knowledge in order to progress toward the enduring understandings.

**Essential knowledge** statements define the required content knowledge associated with each learning objective assessed on the AP Exam.

**Boundary statements** provide guidance to teachers regarding the content boundaries on the AP Physics courses. Boundary statements appear at the end of essential knowledge statements where appropriate.

THIS PAGE IS INTENTIONALLY LEFT BLANK.

**AP PHYSICS C: ELECTRICITY  
AND MAGNETISM**

**UNIT 8**

**Electric  
Charges,  
Fields, and  
Gauss's Law**



**15–25%**  
AP EXAM WEIGHTING



**~12/~24**  
CLASS PERIODS

---

The icon consists of a white circle containing a blue square with the letters 'AP' in white. Below the square is a small black monitor-like shape with two vertical lines representing a stand.

Remember to go to [AP Classroom](#) to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

### Progress Check 8

**Multiple-choice: ~18 questions**

**Free-response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# Electric Charges, Fields, and Gauss’s Law



## Developing Understanding

### ESSENTIAL QUESTIONS

- Why does hair stand up after brushing it with a plastic comb?
- How does a charged rubber rod bend a stream of water?
- How might it be possible to get a balloon to stick to the wall?
- Why don’t cell phones work in concrete buildings?
- Why can a bird land safely on a high voltage wire?

In Unit 8, students will begin the study of electric force, which is exerted on all objects with a property called charge. The electric force, in contrast to gravitational force, is one of attraction or repulsion and, therefore, leads to different effects on objects. This knowledge will help students understand the role electrostatics plays in common devices such as photocopiers, defibrillators, and printers, as well as television, radio, and radar industries. In the units that follow, students will apply their knowledge of electric charges and force to electric circuits, and how the motion of electric charges creates magnetic fields.

## Building the Science Practices

1.A 1.C 2.A

Unit 8 provides multiple opportunities for students to create and use visual representations (**1.A** and **1.C**) to demonstrate an understanding of the relationships between the variables that describe the motion of objects or systems. Unit 8 will also reinforce the importance of demonstrating consistency between different graphical representations of the same physical situation. Being able to identify, create, and use graphs that represent the same physical situation demonstrates a deeper understanding of concepts than simply creating or using one representation. As students incorporate these skills in Unit 8, their abilities to identify consistencies between graphs as well as to create graphs that are consistent with each other will improve with continued practice. Lastly, throughout the unit, students will be challenged to identify which fundamental law, definition, and/or mathematical relationship will apply to a given scenario. Selecting the appropriate relationship and subsequent solution technique are critical problem-solving skills that should be given space to be developed as a first step towards deriving equations (**2.A**) to represent physical scenarios.

## Preparing for the AP Exam

The first question on the AP Physics C: Electricity and Magnetism Exam—the Mathematical Routines (MR) question—focuses on assessing students’ ability to create and use mathematical models and representations. Students will be required derive an expression for a physical quantity which may culminate in a numerical calculation. Students will expand on these mathematical skills by either creating a representation of the scenario at hand, or making and justifying claims about that same scenario. While Unit 8 offers content perfect for practicing the MR question, the MR question on the AP Physics C: Electricity and Magnetism Exam can pull content from any of the six units of the course.



## UNIT AT A GLANCE

Topics	Suggested Skills
<b>8.1 Electric Charge and Electric Force</b>	<p><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</p> <p><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</p> <p><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>8.2 Conservation of Electric Charge and the Process of Charging</b>	<p><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>
<b>8.3 Electric Fields</b>	<p><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</p> <p><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>8.4 Electric Fields of Charge Distributions</b>	<p><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>

*continued on next page*

**UNIT AT A GLANCE** *(cont'd)*

Topics	Suggested Skills
<b>8.5 Electric Flux</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>8.6 Gauss's Law</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</li><li><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</li></ul>



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 8.  
Review the results in class to identify and address any student misunderstandings.

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	8.1	<b>Desktop Experiment Tasks</b> Give two pith balls some amount of charge (assumed to be equal charges) and hang the balls from string near enough to each other that they exert a repulsive force on each other. Have students measure the angle their strings make and use this information to determine the charge on the pith balls. Also have them determine what fraction of the pith ball's electrons were lost/gained assuming one electron for every $3.3 \times 10^{-27}$ kg of mass.
2	8.1	<b>Qualitative Reasoning</b> Have students consider a situation where two metal spheres (one heavy, one light) have unequal magnitude, opposite charges, and are set at rest near each other in space. Then, have students draw acceleration versus time and velocity versus time graphs for the times when the light sphere attracts, collides elastically with, and then repels from the heavier sphere.
3	8.2	<b>Desktop Experiment Tasks</b> Have students place sticky tape, sticky side down, on the tops of their desks. Then, have students place a second piece of tape on top of the first and then a third piece on top of the second. Have students quickly remove pieces 2 and 3 simultaneously from the desk. Then, quickly separate pieces 2 and 3 from each other. Have students predict whether the pieces of tape will be charged and if they will have different or similar charges.
4	8.3	<b>Desktop Experiment Tasks</b> Have students play the PhET simulation "Electric Field Hockey." Have a competition to see which group of students can score a goal with the least number of charges guiding the test particle.
5	8.6	<b>Create a Plan</b> Have students research the electric field strength and direction at ground level on Earth. Next, have them use Gauss's Law to determine the net charge on Earth.

## TOPIC 8.1

**Electric Charge and Electric Force**

## SUGGESTED SKILLS

**1.A**

Create diagrams, tables, charts, or schematics to represent physical situations.

**2.B**

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

**2.D**

Predict new values or factors of change of physical quantities using functional dependence between variables.

**3.B**

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

**Required Course Content****LEARNING OBJECTIVE****8.1.A**

Describe the electric force that results from the interactions between charged objects or systems.

**ESSENTIAL KNOWLEDGE****8.1.A.1**

Charge is a fundamental property of all matter.

**8.1.A.1.i**

Charge is a scalar quantity and is described as positive or negative.

**8.1.A.1.ii**

The magnitude of the charge of a single electron or proton, the elementary charge  $e$ , can be considered to be the smallest indivisible amount of charge.

**8.1.A.1.iii**

The charge of an electron is  $-e$  and the charge of a proton is  $+e$ , and a neutron has no electric charge.

**8.1.A.1.iv**

A point charge is a model in which the physical size of a charged object or system is negligible in the context of the situation being analyzed.

**8.1.A.2**

Coulomb's law describes the electrostatic force between two charged objects as directly proportional to the magnitude of each of the charges and inversely proportional to the square of the distance between the objects.

Relevant equation:

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2} = k \frac{|q_1q_2|}{r^2}$$

*continued on next page*

## LEARNING OBJECTIVE

**8.1.A**

Describe the electric force that results from the interactions between charged objects or systems.

**8.1.B**

Describe the electric and gravitational forces that result from interactions between charged objects with mass.

**8.1.C**

Describe the electric permittivity of a material or medium.

## ESSENTIAL KNOWLEDGE

**8.1.A.3**

The direction of the electrostatic force depends on the signs of the charges of the interacting objects and is along the line of separation between the objects.

**8.1.A.3.i**

Two objects with charges of the same sign exert repulsive forces on each other.

**8.1.A.3.ii**

Two objects with charges of opposite signs exert attractive forces on each other.

**8.1.A.4**

Electric forces are responsible for some of the macroscopic properties of objects in everyday experiences. However, the large number of particle interactions that occur make it more convenient to treat everyday forces in terms of nonfundamental forces called contact forces, such as normal force, friction, and tension.

**8.1.B.1**

Electrostatic forces can be attractive or repulsive, while gravitational forces are always attractive.

**8.1.B.2**

For any two objects that have mass and electric charge, the magnitude of the gravitational force is usually much smaller than the magnitude of the electrostatic force.

**8.1.B.3**

Gravitational forces dominate at larger scales even though they are weaker than electrostatic forces, because systems at large scales tend to be electrically neutral.

**8.1.C.1**

Electric permittivity is a measurement of the degree to which a material or medium is polarized in the presence of an electric field.

**8.1.C.2**

Electric polarization can be modeled as the induced rearrangement of electrons by an external electric field, resulting in a separation of positive and negative charges within a material or medium.

**8.1.C.3**

Free space has a constant value of electric permittivity,  $\epsilon_0$ , that appears in physical relationships.

*continued on next page*

## LEARNING OBJECTIVE

### 8.1.C

Describe the electric permittivity of a material or medium.

## ESSENTIAL KNOWLEDGE

### 8.1.C.4

The permittivity of matter has a value different from that of free space that arises from the matter's composition and arrangement.

#### 8.1.C.4.i

In a given material, electric permittivity is determined by the ease with which electrons can change configurations within the material.

#### 8.1.C.4.ii

Conductors are made from electrically conducting materials in which charge carriers move easily; insulators are made from electrically nonconducting materials in which charge carriers cannot move easily.

## BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism only expects students to make calculations of the electric force between four or fewer interacting charged objects or systems. The analysis of the resulting electric force from more charges is allowed in situations of high symmetry. Note that students are expected to calculate the electric fields of charge distributions, as described in Topics 8.4 and 8.6.*

## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## TOPIC 8.2

# Conservation of Electric Charge and the Process of Charging

## Required Course Content

### LEARNING OBJECTIVE

## 8.2.A

Describe the behavior of a system using conservation of charge.

### ESSENTIAL KNOWLEDGE

## 8.2.A.1

The net charge or charge distribution of a system can change in response to the presence of, or changes in, the net charge or charge distribution of other systems.

## 8.2.A.1.i

The net charge of a system can change due to friction or contact between systems.

## 8.2.A.1.ii

Induced charge separation occurs when the electrostatic force between two systems alters the distribution of charges within the systems, resulting in the polarization of one or both systems.

## 8.2.A.1.iii

Induced charge separation can occur in neutral systems.

## 8.2.A.2

Any change to a system's net charge is due to a transfer of charge between the system and its surroundings.

## 8.2.A.2.i

The charging of a system typically involves the transfer of electrons to and from the system.

## 8.2.A.2.ii

The net charge of a system will be constant unless there is a transfer of charge to or from the system.

## 8.2.A.3

Grounding involves electrically connecting a charged object to a much larger and approximately neutral system (e.g., Earth).

## TOPIC 8.3

# Electric Fields

### Required Course Content

#### LEARNING OBJECTIVE

##### 8.3.A

Describe the electric field produced by a charged object or configuration of point charges.

#### ESSENTIAL KNOWLEDGE

##### 8.3.A.1

Electric fields may originate from charged objects.

##### 8.3.A.2

The electric field at a given point is the ratio of the electric force exerted on a test charge at the point to the charge of the test charge.

*Relevant equation:*

$$\vec{E} = \frac{\vec{F}_E}{q}$$

##### 8.3.A.2.i

A test charge is a point charge of small enough magnitude such that its presence does not significantly affect an electric field in its vicinity.

##### 8.3.A.2.ii

An electric field points away from isolated positive charges and toward isolated negative charges.

##### 8.3.A.2.iii

The electric force exerted on a positive test charge by an electric field is in the same direction as the electric field.

#### SUGGESTED SKILLS

##### 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

##### 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

##### 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

##### 3.A

Create experimental procedures that are appropriate for a given scientific question.

##### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

*continued on next page*



## LEARNING OBJECTIVE

**8.3.A**

Describe the electric field produced by a charged object or configuration of point charges.

**8.3.B**

Describe the electric field generated by charged conductors or insulators.

## ESSENTIAL KNOWLEDGE

**8.3.A.3**

The electric field is a vector quantity and can be represented in space using vector field maps.

**8.3.A.3.i**

The net electric field at a given location is the vector sum of individual electric fields created by nearby charged objects.

**8.3.A.3.ii**

Electric field maps use vectors to depict the magnitude and direction of the electric field at many locations within a given region.

**8.3.A.3.iii**

Electric field line diagrams are simplified models of electric field maps and can be used to determine the relative magnitude and direction of the electric field at any position in the diagram.

**8.3.B.1**

While in electrostatic equilibrium, the excess charge of a conductor is distributed on the surface of the conductor, and the electric field within the conductor is zero.

**8.3.B.1.i**

At the surface of a charged conductor, the electric field is perpendicular to the surface.

**8.3.B.1.ii**

The electric field outside an isolated sphere with spherically symmetric charge distribution is the same as the electric field due to a point charge with the same net charge as the sphere located at the center of the sphere.

**8.3.B.2**

While in electrostatic equilibrium, the excess charge of an insulator is distributed throughout the interior of the insulator as well as at the surface, and the electric field within the insulator may have a nonzero value.

## TOPIC 8.4

# Electric Fields of Charge Distributions

## Required Course Content

### LEARNING OBJECTIVE

**8.4.A**

Describe the electric field resulting from a given charge distribution.

### ESSENTIAL KNOWLEDGE

**8.4.A.1**

Expressions for the electric field of specified charge distributions can be found using integration and the principle of superposition.

*Relevant equation:*

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

**8.4.A.2**

Symmetry considerations of certain charge distributions can simplify analysis of the electric field resulting from those charge distributions.

### BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism only expects students to use calculus to find the electric field resulting from the following charge distributions and locations: an infinitely long, uniformly charged wire or cylinder at a distance from its central axis, a thin ring of charge at a location along the axis of the ring, a semicircular arc or part of a semicircular arc at its center, and a finite wire or line charge at a point collinear with the line charge or at a location along its perpendicular bisector.*

### SUGGESTED SKILLS

**1.C**

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

**2.A**

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

**2.C**

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

**3.C**

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## TOPIC 8.5

## Electric Flux

## Required Course Content

## LEARNING OBJECTIVE

## 8.5.A

Describe the electric flux through an arbitrary area or geometric shape.

## ESSENTIAL KNOWLEDGE

## 8.5.A.1

Flux describes the amount of a given quantity that passes through a given area.

## 8.5.A.2

For an electric field  $\vec{E}$  that is constant across an area  $\vec{A}$ , the electric flux through the area is defined as

$$\Phi_E = \vec{E} \cdot \vec{A}.$$

## 8.5.A.2.i

The direction of the area vector is defined as perpendicular to the plane of the surface and outward from a closed surface.

## 8.5.A.2.ii

The sign of flux is given by the dot product of the electric field vector and the area vector.

## 8.5.A.3

The total electric flux passing through a surface is defined by the surface integral of the electric field over the surface.

*Relevant equation:*

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

## TOPIC 8.6

# Gauss's Law

### SUGGESTED SKILLS

#### 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

#### 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

#### 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

#### 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

## Required Course Content

### LEARNING OBJECTIVE

#### 8.6.A

Describe the properties of a charge distribution by applying Gauss's law.

### ESSENTIAL KNOWLEDGE

#### 8.6.A.1

Gauss's law relates electric flux through a Gaussian surface to the charge enclosed by that surface.

*Relevant equations:*

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

#### 8.6.A.2

A Gaussian surface is a three-dimensional, closed surface.

#### 8.6.A.3

The total electric flux through a Gaussian surface is independent of the size of the Gaussian surface if the amount of enclosed charge remains constant.

#### 8.6.A.4

Gaussian surfaces are typically constructed such that the electric field generated by the enclosed charge is either perpendicular or parallel to different regions of the Gaussian surface, resulting in a simplified surface integral.

#### 8.6.A.5

If a function of charge density is given for a charge distribution, the total charge can be determined by integrating the charge density over the length (one dimension), area (two dimensions), or volume (three dimensions) of the charge distribution. For example:

$$Q_{\text{total}} = \int \rho(\vec{r}) dV$$

*continued on next page*

## LEARNING OBJECTIVE

## 8.6.A

Describe the properties of a charge distribution by applying Gauss's law.

## ESSENTIAL KNOWLEDGE

## 8.6.A.6

Maxwell's equations are the collection of equations that fully describe electromagnetism. Gauss's law is Maxwell's first equation.

Relevant equation:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

## BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism only expects students to quantitatively apply Gauss's law to point charges and charge distributions that have spherical, cylindrical, or planar symmetry.*

**AP PHYSICS C: ELECTRICITY  
AND MAGNETISM**

**UNIT 9**

**Electric  
Potential**



**10–20%**  
AP EXAM WEIGHTING



**~10/~20**  
CLASS PERIODS

---

The icon consists of a white circle containing a blue square with the letters 'AP' in white. Below the square is a small blue monitor icon with two lines representing a screen and a base.

Remember to go to [AP Classroom](#) to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

### Progress Check 9

**Multiple-choice: ~18 questions**

**Free-response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# Electric Potential



## Developing Understanding

### ESSENTIAL QUESTIONS

- What is the difference between electric potential and electric potential energy?
- Why do different voltage batteries exist?
- What is the difference between a 1.5 V AA battery and a 9 V battery?
- Why is a car battery more dangerous than a 9 V battery?
- Why are high voltage power lines dangerous?

In Unit 9, students are introduced to the concept of electric potential, which is another way to describe interactions between charged systems. A thorough understanding of the relationship between electric potential and electrical energy will support student’s ability to analyze physical scenarios, such as the interaction between two charged objects, using the concept of energy. Students will begin to appreciate the real-world implications, including energy storage and transfer. Throughout the entire course, students will apply their knowledge of electric potential and potential energy to a wide array of scenarios to describe how electrical energy powers our world.

## Building the Science Practices

1.A 1.C 2.A 3.C

Throughout this unit, there are many opportunities for students to create graphs (1.C) that may include electric potential or potential energy of a system as either a function of position or time for a single scenario, as well as to make connections between physics concepts based on these graphs.

In Unit 9, as in other units in AP Physics C: Electricity and Magnetism, practice creating and using models to represent physical scenarios (1.A) and then translating the information presented in these models into other representations, such as symbolic expressions (2.A), can help students justify or support claims about the electric potential of a system (3.C).

## Preparing for the AP Exam

The second free-response question on the AP Physics C: Electricity and Magnetism Exam—the Translation Between Representations (TBR) question—requires students to create graphical and verbal models of scenarios, as well as compare these models to mathematical representations of the same situation. Similar in nature to the Qualitative/Quantitative Translation (QQT), the TBR involves creating multiple representations and describing the relationships between those representations; however, the types of representations being compared in the TBR differ from those in the QQT. In the TBR, a student might be asked to sketch an equipotential diagram from an electric field map. The student might then be asked to create an energy diagram for a point charge moving inside the region with an electric field. Lastly, the student might be asked to make connections between the two representations, justifying how the representations are consistent with each other. While Unit 9 content provides especially good practice for the TBR, content from any unit may be included in this free-response question on the AP Exam.



## UNIT AT A GLANCE

Topics	Suggested Skills
<b>9.1 Electric Potential Energy</b>	<p><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>
<b>9.2 Electric Potential</b>	<p><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>9.3 Conservation of Electric Energy</b>	<p><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</p> <p><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 9.  
Review the results in class to identify and address any student misunderstandings.

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	9.2	<b>Changing Representations</b> Have students use the Charges and Fields PhET or the applet at <a href="http://flashphysics.org/electricField.html">flashphysics.org/electricField.html</a> to investigate electric field and potential (and their relationship) in the vicinity of equal or unequal two- or three-charge systems.
2	9.2	<b>Desktop Experiment Tasks</b> Connect two electrodes to a 9 V battery and immerse them in a plastic pan of water that is less than 1 cm deep. Use a voltmeter (negative connected to the negative of the battery) to probe the electric potential at various points in the water. Have students construct an electric potential isoline map and estimate the strength of the electric field at various locations.

## SUGGESTED SKILLS

## 1.C

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## TOPIC 9.1

# Electric Potential Energy

## Required Course Content

### LEARNING OBJECTIVE

## 9.1.A

Describe the electric potential energy of a system.

### ESSENTIAL KNOWLEDGE

## 9.1.A.1

The electric potential energy of a system of two point charges equals the amount of work required for an external force to bring the point charges to their current positions from infinitely far away.

## 9.1.A.2

The general form for the electric potential energy between two charged objects is given by the equation

$$U_E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = k \frac{q_1 q_2}{r}.$$

## 9.1.A.3

The total electric potential energy of a system can be determined by finding the sum of the electric potential energies of the individual interactions between each pair of charged objects in the system.

## TOPIC 9.2

# Electric Potential

### SUGGESTED SKILLS

#### 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

#### 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

#### 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

#### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## Required Course Content

### LEARNING OBJECTIVE

#### 9.2.A

Describe the electric potential due to a configuration of charged objects.

### ESSENTIAL KNOWLEDGE

#### 9.2.A.1

Electric potential describes the electric potential energy per unit charge at a point in space.

#### 9.2.A.2

Expressions for the electric potential of charge distributions can be found using integration and the principle of superposition.

*Relevant equation:*

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

#### 9.2.A.2.i

The electric potential for single point charge is

$$V = \frac{q}{4\pi\epsilon_0 r}$$

#### 9.2.A.2.ii

The electric potential due to multiple point charges can be determined by the principle of scalar superposition of the electric potential due to each of the point charges.

*Relevant equation:*

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

#### 9.2.A.3

The electric potential difference between two points is the change in electric potential energy per unit charge when a test charge is moved between the two points.

*Relevant equation:*

$$\Delta V = \frac{\Delta U_E}{q}$$

continued on next page

## LEARNING OBJECTIVE

## 9.2.A

Describe the electric potential due to a configuration of charged objects.

## ESSENTIAL KNOWLEDGE

## 9.2.A.4

Electric potential difference may also result from chemical processes that cause positive and negative charges to separate, such as in a battery.

## BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism only expects students to use calculus to find the electric potential resulting from the following charge distributions and locations: an infinitely long, uniformly charged wire or cylinder at a distance from its central axis, a thin ring of charge at a location along the axis of the ring, a semicircular arc or part of a semicircular arc at its center, and a finite wire or line charge at a point collinear with the line charge or at a location along its perpendicular bisector.*

## 9.2.B

Describe the relationship between electric potential and electric field.

## 9.2.B.1

The value of an electric field component in any direction at a given location is equal to the negative of the spatial rate of change in electric potential at that location.

Relevant equation:

$$E_x = -\frac{dV}{dx}$$

## 9.2.B.2

The change in electric potential between two points can be determined by integrating the dot product of the electric field and the displacement along the path connecting the points.

Relevant equation:

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{r}$$

## 9.2.B.3

Electric field vector maps and equipotential lines are tools to describe the field produced by a charge or configuration of charges and can be used to predict the motion of charged objects in the field.

## 9.2.B.3.i

Equipotential lines represent lines of equal electric potential. These lines are also referred to as isolines of electric potential.

*continued on next page*

## LEARNING OBJECTIVE

### 9.2.B

Describe the relationship between electric potential and electric field.

## ESSENTIAL KNOWLEDGE

### 9.2.B.3.ii

Isolines are perpendicular to electric field vectors. An isoline map of electric potential can be constructed from an electric field vector map, and an electric field map may be constructed from an isoline map.

### 9.2.B.3.iii

An electric field vector points in the direction of decreasing potential.

### 9.2.B.3.iv

There is no component of an electric field along an isoline.

## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

## 3.A

Create experimental procedures that are appropriate for a given scientific question.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## TOPIC 9.3

# Conservation of Electric Energy

## Required Course Content

### LEARNING OBJECTIVE

## 9.3.A

Describe changes in a system due to a difference in electric potential between two locations.

### ESSENTIAL KNOWLEDGE

## 9.3.A.1

When a charged object moves between two locations with different electric potentials, the resulting change in the electric potential energy of the object-field system is given by the following equation.

*Relevant equation:*

$$\Delta U_E = q\Delta V$$

## 9.3.A.2

The movement of a charged object between two points with different electric potentials results in a change in kinetic energy of the object consistent with the conservation of energy.

**AP PHYSICS C: ELECTRICITY  
AND MAGNETISM**

**UNIT 10**

**Conductors  
and  
Capacitors**



**10–15%**  
AP EXAM WEIGHTING



**~8/~16**  
CLASS PERIODS

---



The icon consists of a white circle containing a blue square with the letters 'AP' in white. Below the square is a small black monitor-like shape with two vertical lines representing a stand.

Remember to go to [AP Classroom](#) to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

### Progress Check 10

**Multiple-choice: ~18 questions**

**Free-response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# Conductors and Capacitors



## Developing Understanding

### ESSENTIAL QUESTIONS

- Why does the little red light on the TV stay on for a second after the TV is turned off?
- How does a camera flash used for photography work?
- How can energy be stored for later use?

In Unit 8, students investigated why all objects have an electric charge. In Unit 10, students will examine how that charge can be stored. Conductors, capacitors, and dielectrics are presented to demonstrate that the ability of charge to move is dependent on the material composition of an object. In electronics, each of these are important based on the type of movement or desired object behavior. Additionally, this unit examines how the behavior of charges is impacted by electric fields. Students can benefit from opportunities (e.g., laboratory investigations or activities) to describe and examine the function of each of these elements, along with capacitors. Knowledge of conductors, capacitors, and dielectrics will prepare students for understanding how electric circuits work in Unit 11 and how they behave when one or more electrical element is altered or modified.

## Building the Science Practices

2.A 3.B 3.C

Translation between models and representations is key in this unit. Students will continue to use models and representations that will help them further analyze systems, the interactions between systems, and how those interactions result in change. Alongside gaining proficiency in the use of specific equations for capacitance, Unit 10 also encourages students to derive new expressions from fundamental principles (2.A) to help them make predictions using functional dependence between variables (2.D). The skills of making claims (3.B) and supporting those claims using evidence (3.C) can be developed through the unit by providing students with opportunities such as having them make predictions about the net charge on a capacitor based on the geometry of the capacitor, and then justifying those predictions with appropriate physics principles.

## Preparing for the AP Exam

In the free-response section of the AP Physics C: Electricity and Magnetism Exam, the fourth question—the Qualitative/Quantitative Translation (QQT) question—requires students to re-express key elements of physical phenomena across multiple representations of the domain. In this question, students demonstrate their ability to analyze and describe a scenario, expressing that description both verbally and mathematically. Using content from any unit, the QQT first requires students to make a claim and provide evidence and reasoning to support their claim without reference to equations. Students are then asked to derive an equation or set of equations to mathematically represent the scenario. Lastly, students are required to make a connection between the claim made in the first part of the question and the equation(s) derived in the second part. Students exposed primarily to numerical problem solving often struggle with the QQT because it requires them to communicate



UNIT  
10

---

## Conductors and Capacitors

---

a conceptual understanding of course content. Having frequent opportunities—including those that present themselves in Unit 10—to translate between different

representations, including equations, diagrams, graphs, and verbal descriptions, can help students prepare for the QQT question.

## UNIT AT A GLANCE

Topics	Suggested Skills
<b>10.1 Electrostatics with Conductors</b>	<p><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>
<b>10.2 Redistribution of Charge between Conductors</b>	<p><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>10.3 Capacitors</b>	<p><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>10.4 Dielectrics</b>	<p><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</p> <p><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</p> <p><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</p> <p><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 10.  
Review the results in class to identify and address any student misunderstandings

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	10.1	<b>Predict and Justify</b> Find an enclosed metal wire mesh container that can fit a neon gas discharge tube. Touch the tube to a tesla coil to show it lighting up. Then, put the tube in the wire mesh and have students predict what will happen (i.e., no amount of effort will cause the tube to light up from the tesla coil). Ask students to explain why they think their prediction will occur and to justify their claim with evidence.
2	10.3	<b>Create a Plan</b> Have students research the electric properties of the ionosphere and Earth's surface and use their research to determine the following: The capacitance of the spherical system consisting of ionosphere and Earth's surface, and the charge and potential difference of the "capacitor."
3	10.3	<b>Desktop Experiment Tasks</b> Have students construct their own capacitor from sheets of aluminum foil and waxed paper, and ask students to predict its capacitance. Then, have students use a capacitance meter to measure the capacitance.
4	10.4	<b>Construct an Argument</b> Have students justify with evidence why it is that pure water has such a high dielectric constant (i.e., the polarity of the molecules). Also have them justify with evidence why impure water results in "leaky" capacitors made from a water dielectric (i.e., the impurities conduct current).
5	10.4	<b>Desktop Experiment Tasks</b> Have students construct a parallel-plate capacitor out of two pieces of foil with a piece of paper between each piece. Have students measure the capacitance with a capacitance meter. Next, have them increase the number of sheets of paper and record data with the purpose of finding the dielectric constant of the paper.

# TOPIC 10.1

## Electrostatics with Conductors

### Required Course Content

#### LEARNING OBJECTIVE

##### 10.1.A

Describe the charge distribution within a conductor.

#### ESSENTIAL KNOWLEDGE

##### 10.1.A.1

An ideal conductor is a material in which electrons are able to move freely.

##### 10.1.A.2

When a conductor is in electrostatic equilibrium, mutual repulsion of excess charge carriers results in those charge carriers residing entirely on the surface of the conductor.

##### 10.1.A.2.i

In a conductor with a negative net charge, excess electrons reside on the surface of the conductor.

##### 10.1.A.2.ii

In a conductor with a positive net charge, the surface becomes deficient in electrons, and can be modeled as if positive charge carriers reside on the surface of the conductor.

##### 10.1.A.3

Excess charges will move to the surface of a conductor to create a state of electrostatic equilibrium within the conductor.

##### 10.1.A.3.i

The time interval over which charges reach electrostatic equilibrium within a conductor is so short as to be negligible.

##### 10.1.A.3.ii

When a conductor reaches electrostatic equilibrium, all points on the surface of the conductor have the same electric potential, and the conductor becomes an equipotential surface.

#### SUGGESTED SKILLS

##### 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

##### 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

##### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

##### 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

*continued on next page*

## LEARNING OBJECTIVE

**10.1.A**

Describe the charge distribution within a conductor.

## ESSENTIAL KNOWLEDGE

**10.1.A.3.iii**

The charge density on the surface of a conductor will be greater where there are points or edges compared to planar areas.

**10.1.A.4**

All excess charges reside on the surface of a conductor, which means there is no net charge in the interior of the conductor, and the electric field is zero within the conductor.

**10.1.A.5**

The electric field is perpendicular to the outer surface of a conductor.

**10.1.A.6**

A conductor can be polarized in the presence of an external electric field. This is a consequence of the conductor remaining an equipotential surface.

**10.1.A.7**

Electrostatic shielding is the process of surrounding an area with a closed, conducting shell to create a region inside the conductor that is free from external electric fields.

## TOPIC 10.2

# Redistribution of Charge Between Conductors

## Required Course Content

### LEARNING OBJECTIVE

**10.2.A**

Describe the movement of charge and the resulting interactions when conductors physically contact each other.

### ESSENTIAL KNOWLEDGE

**10.2.A.1**

When conductors are in electrical contact, charges will be redistributed such that the surfaces of each conductor are at the same electric potential.

**10.2.A.2**

Ground is an idealized reference point that has zero electric potential and can absorb or provide an infinite amount of charge without changing its electric potential.

**10.2.A.3**

Charge can be induced on a conductor by grounding the conductor in the presence of an external electric field.

**SUGGESTED SKILLS****1.A**

Create diagrams, tables, charts, or schematics to represent physical situations.

**2.A**

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

**2.C**

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

**3.B**

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.



## SUGGESTED SKILLS

## 1.C

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## TOPIC 10.3

# Capacitors

### Required Course Content

#### LEARNING OBJECTIVE

## 10.3.A

Describe the physical properties of a parallel-plate capacitor.

#### ESSENTIAL KNOWLEDGE

## 10.3.A.1

A parallel-plate capacitor consists of two separated parallel conducting surfaces that can hold equal amounts of charge with opposite signs.

## 10.3.A.2

Capacitance relates the magnitude of the charge stored on each plate to the electric potential difference created by the separation of those charges.

*Relevant equation:*

$$C = \frac{Q}{\Delta V}$$

## 10.3.A.2.i

The capacitance of a capacitor depends only on the physical properties of the capacitor, such as the capacitor's shape and the material used to separate the plates.

## 10.3.A.2.ii

The capacitance of a parallel-plate capacitor is proportional to the area of one of its plates and inversely proportional to the distance between its plates. The constant of proportionality is the product of the dielectric constant,  $\kappa$ , of the material between the plates and the electric permittivity of free space,  $\epsilon_0$ .

*Relevant equation:*

$$C = \frac{\kappa \epsilon_0 A}{d}$$

*continued on next page*

## LEARNING OBJECTIVE

### 10.3.A

Describe the physical properties of a parallel-plate capacitor.

## ESSENTIAL KNOWLEDGE

### 10.3.A.3

The electric field between two charged parallel plates with uniformly distributed electric charge, such as in a parallel-plate capacitor, is constant in both magnitude and direction, except near the edges of the plates.

#### 10.3.A.3.i

The magnitude of the electric field between two charged parallel plates, where the plate separation is much smaller than the dimensions of the plates, can be determined by applying Gauss's law and the principle of superposition.

*Derived equation:*

$$E = \frac{Q}{\epsilon_0 A}$$

#### 10.3.A.3.ii

The electric field is proportional to the surface charge density on either plate of the capacitor.

#### 10.3.A.3.iii

A charged particle between two oppositely charged parallel plates undergoes constant acceleration, and therefore its motion shares characteristics with the projectile motion of an object with mass in the gravitational field near Earth's surface.

### 10.3.A.4

The electric potential energy stored in a capacitor is equal to the work done by an external force to separate that amount of charge on the capacitor.

### 10.3.A.5

The electric potential energy stored in a capacitor is described by the equation

$$U_c = \frac{1}{2} Q \Delta V .$$

## BOUNDARY STATEMENT

*While other shapes are also able to separate charges, AP Physics C: Electricity & Magnetism only expects the quantitative analysis and description of parallel-plate capacitors, concentric spherical capacitors, and coaxial cylindrical capacitors.*

## SUGGESTED SKILLS

## 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

## 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

## 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

## 3.A

Create experimental procedures that are appropriate for a given scientific question.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## TOPIC 10.4

# Dielectrics

### Required Course Content

#### LEARNING OBJECTIVE

## 10.4.A

Describe how a dielectric inserted between the plates of a capacitor changes the properties of the capacitor.

#### ESSENTIAL KNOWLEDGE

## 10.4.A.1

In a dielectric material, electric charges are not as free to move as they are in a conductor. Instead, the material becomes polarized in the presence of an external electric field.

## 10.4.A.2

The dielectric constant of a material relates the electric permittivity of that material to the permittivity of free space.

*Relevant equation:*

$$\kappa = \frac{\epsilon}{\epsilon_0}$$

## 10.4.A.3

The electric field created by a polarized dielectric is opposite in direction to the external field.

## 10.4.A.4

The electric field between the plates of an isolated parallel-plate capacitor decreases when a dielectric is placed between the plates.

*Derived equation:*

$$\kappa = \frac{E_0}{E}$$

## 10.4.A.5

The insertion of a dielectric into a capacitor may change the capacitance of the capacitor.

*Derived equation:*

$$C = \kappa C_0$$

# AP PHYSICS C: ELECTRICITY AND MAGNETISM

## UNIT 11

# Electric Circuits



**15–25%**  
AP EXAM WEIGHTING



**~12/~24**  
CLASS PERIODS

---

The icon consists of a white circle containing a blue square with the letters 'AP' in white. Below the square is a small blue monitor-like shape with two horizontal lines representing a screen and a base.

Remember to go to [AP Classroom](#) to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

### Progress Check 11

**Multiple-choice: ~24 questions**

**Free-response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# Electric Circuits



## Developing Understanding

### ESSENTIAL QUESTIONS

- How does a house's wiring design account for a flipped circuit breaker to cause electricity to go off in some rooms but stay on in other rooms?
- Why do warming bulbs take several minutes to shine brightly?
- Why does the electric company charge by kilowatt-hour instead of electrons used?
- How does touching a conductor to a capacitor before removing it from a circuit protect you?

Whether or not they're aware, students interact with electric circuits regularly through charging their phones, powering up their laptops, or simply switching on a light. Unit 11 serves to illuminate how, and why, simple electronic devices such as lightbulbs and household wiring function by exploring the nature and importance of electric currents, circuits, and resistance. Through activities and lab investigations, students will have opportunities to relate knowledge across the course by using the electrical components they learned about in Unit 10 and will come to discover in Unit 11 to create, modify, and analyze circuits. Students will also analyze the relationships that exist between current, resistance, and power, in addition to exploring and applying Ohm's Law and Kirchhoff's Rules.

## Building the Science Practices

1.B 2.B 2.D 3.A

Inquiry learning and critical thinking and problem solving skills are best developed when scientific inquiry experiences are designed and implemented with increasing student involvement. In Unit 11, students can be asked to practice collecting data and determining appropriate experimental procedures to answer scientific questions (3.A). For example, students can be asked to analyze the structure of a circuit by providing a written explanation of observations for light bulbs that turn on or off as various switches are opened or closed.

Once students have designed a procedure and have collected data, they can practice analyzing that data by plotting linearized graphs and using the best fit line to the plotted data to make claims about the physical scenario (1.B, 2.B, 2.D).

## Preparing for the AP Exam

The third question in the free-response section of the AP Physics C: Electricity and Magnetism Exam is the Experimental Design and Analysis (LAB) question. In this question, students will be required to justify their selection of the kind of data needed and then design a plan to collect these data. Because students often struggle with knowing where to start when designing an experiment, they will benefit from scaffolded opportunities to determine the data needed to answer a scientific question. Also in the lab question, students will also be required to derive relevant equations, linearize and analyze data. Practicing designing experiments, performing data analysis, and discussing sources of error throughout the course can help students prepare for and be successful on the Experimental Design and Analysis (LAB) question.

## UNIT AT A GLANCE

Topics	Suggested Skills
<b>11.1 Electric Current</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>11.2 Simple Circuits</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li><li><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li></ul>
<b>11.3 Resistance, Resistivity, and Ohm's Law</b>	<ul style="list-style-type: none"><li><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</li><li><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</li><li><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</li><li><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>11.4 Electric Power</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>11.5 Compound Direct Current Circuits</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li></ul>

*continued on next page*

**UNIT AT A GLANCE** *(cont'd)*

Topics	Suggested Skills
<b>11.6 Kirchhoff's Loop Rule</b>	<p><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>11.7 Kirchhoff's Junction Rule</b>	<p><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</p> <p><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</p> <p><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>
<b>11.8 Resistor–Capacitor (RC) Circuits</b>	<p><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</p> <p><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 11.  
Review the results in class to identify and address any student misunderstandings.



## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	11.3	<b>Desktop Experiment Tasks</b> Give students water, modeling clay, or a related substance and ask them to determine whether the substance is ohmic by applying various voltages and measuring the resulting current.
2	11.4	<b>Construct an Argument</b> Have students justify with evidence why a small $1\ \Omega$ resistor can only handle a small amount of power (such as $1/4\ \text{W}$ ) and why a large $1\ \text{k}\Omega$ resistor can handle a large power (such as $30\ \text{W}$ ). Have students justify with evidence why computer processors have "heat sinks" attached to them.
3	11.5	<b>Desktop Experiment Tasks</b> Give students a small battery and a handful of resistors. Challenge them to put together a circuit where the current delivered by the battery is a certain value. Require students to arrange the resistors in parallel and series to acquire the given current.
4	11.6	<b>Changing Representations</b> Have students solve a typical multi-loop circuit problem with batteries and resistors. Then, have students construct a representation for each possible loop that visually shows Kirchhoff's Loop Rule and a representation for each junction that visually shows Kirchhoff's Junction Rule.
5	11.8	<b>Desktop Experiment Tasks</b> Have students use a known capacitor charged and connected directly to a voltmeter to determine the voltmeter's (high) internal resistance. This is done by taking voltage versus time data as the capacitor discharges through the meter and using the data to find the time constant $RC$ , then $R$ .

## TOPIC 11.1

# Electric Current

### Required Course Content

#### LEARNING OBJECTIVE

##### 11.1.A

Describe the movement of electric charges through a medium.

#### ESSENTIAL KNOWLEDGE

##### 11.1.A.1

Current is the rate at which charge passes through a cross-sectional area of a wire.

*Relevant equation:*

$$I = \frac{dq}{dt}$$

##### 11.1.A.1.i

Current within a conductor consists of charge carriers traveling through the conductor with an average drift velocity.

*Relevant equation:*

$$I = nqv_dA$$

##### 11.1.A.1.ii

Electric charge moves in a circuit in response to an electric potential difference, sometimes referred to as electromotive force, or emf ( $\mathcal{E}$ ).

##### 11.1.A.1.iii

If the current is zero in a section of wire, the net motion of charge carriers in the wire is also zero, although individual charge carriers will not have zero speed.

##### 11.1.A.2

Current density is the flow of charge per unit area.

*Relevant equation:*

$$I = \int \vec{J} \cdot d\vec{A}$$

#### SUGGESTED SKILLS

##### 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

##### 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

##### 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

##### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

*continued on next page*

## LEARNING OBJECTIVE

## 11.1.A

Describe the movement of electric charges through a medium.

## ESSENTIAL KNOWLEDGE

## 11.1.A.2.i

Current density is related to the motion of the charge carriers within a conductor.

*Relevant equation:*

$$\vec{J} = nq\vec{v}_d$$

## 11.1.A.2.ii

Current density is a vector quantity.

## 11.1.A.2.iii

A potential difference across a conductor creates an electric field within the conductor that is proportional to the resistivity of the conductor and the current density.

*Relevant equation:*

$$\vec{E} = \rho\vec{J}$$

## 11.1.A.3

If a function of current density is given, the total current can be determined by integrating the current density over the area.

*Derived equation:*

$$I_{\text{tot}} = \int \vec{J}(r) \cdot d\vec{A}$$

## 11.1.A.4

Although current is a scalar quantity, it does have a direction. Because its direction is relative to the current carrier and not space, current does not obey the laws of vector addition and has no vector components.

## 11.1.A.4.i

The direction of conventional current is chosen to be the direction in which positive charge would move.

## 11.1.A.4.ii

In common circuits, the current is actually due to the movement of electrons (negative charge carriers).

## TOPIC 11.2

# Simple Circuits

### Required Course Content

#### LEARNING OBJECTIVE

**11.2.A**

Describe the behavior of a circuit.

#### ESSENTIAL KNOWLEDGE

**11.2.A.1**

A circuit is composed of electrical loops, which can include wires, batteries, resistors, lightbulbs, capacitors, inductors, switches, ammeters, and voltmeters.

**11.2.A.2**

A closed electrical loop is a closed path through which charges may flow.

**11.2.A.2.i**

A closed circuit is one in which charges would be able to flow.

**11.2.A.2.ii**

An open circuit is one in which charges would not be able to flow.

**11.2.A.2.iii**

A short circuit is one in which charges would be able to flow with no change in potential difference.

**11.2.A.3**

A single circuit element may be part of multiple electrical loops.

**11.2.A.4**

Circuit schematics are representations used to describe and analyze electric circuits.

**11.2.A.4.i**

The properties of an electric circuit are dependent on the physical arrangement of its constituent elements.

#### SUGGESTED SKILLS

**1.A**

Create diagrams, tables, charts, or schematics to represent physical situations.

**2.C**

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

**3.B**

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

**3.C**

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

*continued on next page*

**LEARNING OBJECTIVE**

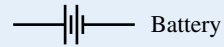
**11.2.A**

Describe the behavior of a circuit.

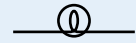
**ESSENTIAL KNOWLEDGE**

**11.2.A.4.ii**

Circuit elements have common symbols that are used to create schematic diagrams. Variable elements are indicated by a diagonal strikethrough arrow across the standard symbol for that element.



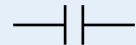
Battery



Bulb



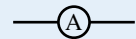
Switch



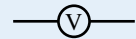
Capacitor



Resistor



Ammeter



Voltmeter



Inductor

**BOUNDARY STATEMENT**

*Unless otherwise specified, all circuit schematic diagrams will be drawn using conventional current.*

## TOPIC 11.3

# Resistance, Resistivity, and Ohm's Law

## Required Course Content

### LEARNING OBJECTIVE

**11.3.A**

Describe the resistance of an object using physical properties of that object.

### ESSENTIAL KNOWLEDGE

**11.3.A.1**

Resistance is a measure of the degree to which an object opposes the movement of electric charge.

**11.3.A.2**

The resistance of a resistor with uniform geometry is proportional to its resistivity and length and is inversely proportional to its cross-sectional area.

*Relevant equation:*

$$R = \frac{\rho \ell}{A}$$

**11.3.A.2.i**

Resistivity is a fundamental property of a material that depends on its atomic and molecular structure and quantifies how strongly the material opposes the motion of electric charge.

**11.3.A.2.ii**

The resistivity of a conductor typically increases with temperature.

**11.3.A.2.iii**

The total resistance of a resistor with uniform geometry, but that is made of a material whose resistivity varies along the length of the resistor, is given by

$$R = \int \frac{\rho(\ell) d\ell}{A}$$

### SUGGESTED SKILLS

**1.B**

Create quantitative graphs with appropriate scales and units, including plotting data.

**2.B**

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

**2.D**

Predict new values or factors of change of physical quantities using functional dependence between variables.

**3.A**

Create experimental procedures that are appropriate for a given scientific question.

**3.B**

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

*continued on next page*

**LEARNING OBJECTIVE****11.3.B**

Describe the electrical characteristics of elements of a circuit.

**ESSENTIAL KNOWLEDGE****11.3.B.1**

Ohm's law relates current, resistance, and potential difference across a conductive element of a circuit.

*Relevant equation:*

$$I = \frac{\Delta V}{R}$$

**11.3.B.1.i**

Materials that obey Ohm's law have constant resistance for all currents and are called ohmic materials.

**11.3.B.1.ii**

The resistivity of an ohmic material is constant regardless of temperature.

**11.3.B.1.iii**

Resistors can also convert electrical energy to thermal energy, which may change the temperature of both the resistor and the resistor's environment.

**11.3.B.1.iv**

The resistance of an ohmic circuit element can be determined from the slope of a graph of the current in the element as a function of the potential difference across the element.

## TOPIC 11.4

# Electric Power

### Required Course Content

#### LEARNING OBJECTIVE

##### 11.4.A

Describe the transfer of energy into, out of, or within an electric circuit, in terms of power.

#### ESSENTIAL KNOWLEDGE

##### 11.4.A.1

The rate at which energy is transferred, converted, or dissipated by a circuit element depends on the current in the element and the electric potential difference across it.

*Relevant equation:*

$$P = I\Delta V$$

*Derived equation:*

$$P = I^2 R = \frac{\Delta V^2}{R}$$

##### 11.4.A.2

The brightness of a lightbulb increases with power, so power can be used to qualitatively predict the brightness of lightbulbs in a circuit.

#### BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism only expects students to analyze the transfer of mechanical and electrical energy, although students should be aware that electrical energy can also be dissipated in the form of thermal energy.*

#### SUGGESTED SKILLS

##### 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

##### 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

##### 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

##### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.



## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## TOPIC 11.5

# Compound Direct Current Circuits

## Required Course Content

### LEARNING OBJECTIVE

## 11.5.A

Describe the equivalent resistance of multiple resistors connected in a circuit.

### ESSENTIAL KNOWLEDGE

## 11.5.A.1

Circuit elements may be connected in series and/or in parallel.

## 11.5.A.1.i

A series connection is one in which any charge passing through one circuit element must proceed through all elements in that connection and has no other path available. The current in each element in series must be the same.

## 11.5.A.1.ii

A parallel connection is one in which charges may pass through one of two or more paths. Across each path, the potential difference is the same.

## 11.5.A.2

A collection of resistors in a circuit may be analyzed as though it were a single resistor with an equivalent resistance  $R_{\text{eq}}$ .

## 11.5.A.2.i

The equivalent resistance of a set of resistors in series is the sum of the individual resistances.

*Relevant equation:*

$$R_{\text{eq},s} = \sum_i R_i$$

*continued on next page*

## LEARNING OBJECTIVE

**11.5.A**

Describe the equivalent resistance of multiple resistors connected in a circuit.

**11.5.B**

Describe a circuit with resistive wires and a battery with internal resistance.

## ESSENTIAL KNOWLEDGE

**11.5.A.2.ii**

The inverse of the equivalent resistance of a set of resistors connected in parallel is equal to the sum of the inverses of the individual resistances.

*Relevant equation:*

$$\frac{1}{R_{\text{eq,p}}} = \sum_i \frac{1}{R_i}$$

**11.5.A.2.iii**

When resistors are connected in parallel, the number of paths available to charges increases, and the equivalent resistance of the group of resistors decreases.

**11.5.B.1**

Ideal batteries have negligible internal resistance. Ideal wires have negligible resistance.

**11.5.B.1.i**

The resistance of wires that are good conductors may normally be neglected, because their resistance is much smaller than that of other elements of a circuit.

**11.5.B.1.ii**

The resistance of wires may only be neglected if the circuit contains other elements that do have resistance.

**11.5.B.1.iii**

The potential difference a battery would supply if it were ideal is the potential difference measured across the terminals when there is no current in the battery and is sometimes referred to as its emf ( $\mathcal{E}$ ).

**11.5.B.2**

The internal resistance of a nonideal battery may be treated as the resistance of a resistor in series with an ideal battery and the remainder of the circuit.

**11.5.B.3**

When there is current in a nonideal battery with internal resistance  $r$ , the potential difference across the terminals of the battery is reduced relative to the potential difference when there is no current in the battery.

*Derived equation:*

$$\Delta V_{\text{terminal}} = \mathcal{E} - Ir$$

*continued on next page*

**LEARNING OBJECTIVE****11.5.C**

Describe the measurement of current and potential difference in a circuit.

**ESSENTIAL KNOWLEDGE****11.5.C.1**

Ammeters are used to measure current at a specific point in a circuit.

**11.5.C.1.i**

Ammeters must be connected in series with the element in which current is being measured.

**11.5.C.1.ii**

Ideal ammeters have zero resistance so that they do not affect the current in the element that they are in series with.

**11.5.C.2**

Voltmeters are used to measure electric potential difference between two points in a circuit.

**11.5.C.2.i**

Voltmeters must be connected in parallel with the element across which potential difference is being measured.

**11.5.C.2.ii**

Ideal voltmeters have infinite resistance so that no charge flows through them.

**11.5.C.3**

Nonideal ammeters and voltmeters will change the properties of the circuit being measured.

**BOUNDARY STATEMENT**

*Unless otherwise stated, all batteries, wires, and meters are assumed to be ideal. Circuits with batteries of different potential differences connected in parallel will not be assessed.*

## TOPIC 11.6

## Kirchhoff's Loop Rule

## SUGGESTED SKILLS

## 1.C

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## Required Course Content

## LEARNING OBJECTIVE

## 11.6.A

Describe a circuit or elements of a circuit by applying Kirchhoff's loop rule.

## ESSENTIAL KNOWLEDGE

## 11.6.A.1

Energy changes in simple electrical circuits may be represented in terms of charges moving through electric potential differences within circuit elements.

*Relevant equation:*

$$\Delta U_E = q\Delta V$$

## 11.6.A.2

Kirchhoff's loop rule is a consequence of the conservation of energy.

## 11.6.A.2.i

Kirchhoff's loop rule states that the sum of potential differences across all circuit elements in a single closed loop must equal zero.

*Relevant equation:*

$$\sum \Delta V = 0$$

## 11.6.A.2.ii

The values of electric potential at points in a circuit can be represented by a graph of electric potential as a function of position within a loop.

## SUGGESTED SKILLS

## 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

## 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

## 3.A

Create experimental procedures that are appropriate for a given scientific question.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## TOPIC 11.7

## Kirchhoff's Junction Rule

## Required Course Content

## LEARNING OBJECTIVE

## 11.7.A

Describe a circuit or elements of a circuit by applying Kirchhoff's junction rule.

## ESSENTIAL KNOWLEDGE

## 11.7.A.1

Kirchhoff's junction rule is a consequence of the conservation of electric charge.

## 11.7.A.2

Kirchhoff's junction rule states that the total amount of charge entering a junction per unit time must equal the total amount of charge exiting that junction per unit time.

*Relevant equation:*

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

## TOPIC 11.8

Resistor-Capacitor  
(RC) Circuits

## Required Course Content

## LEARNING OBJECTIVE

## 11.8.A

Describe the equivalent capacitance of multiple capacitors.

## ESSENTIAL KNOWLEDGE

## 11.8.A.1

A collection of capacitors in a circuit may be analyzed as though it was a single capacitor with an equivalent capacitance  $C_{\text{eq}}$ .

## 11.8.A.1.i

The inverse of the equivalent capacitance of a set of capacitors connected in series is equal to the sum of the inverses of the individual capacitances.

Relevant equation:

$$\frac{1}{C_{\text{eq},s}} = \sum_i \frac{1}{C_i}$$

## 11.8.A.1.ii

The equivalent capacitance of a set of capacitors in series is less than the capacitance of the smallest capacitor.

## 11.8.A.1.iii

The equivalent capacitance of a set of capacitors in parallel is the sum of the individual capacitances.

Relevant equation:

$$C_{\text{eq},p} = \sum_i C_i$$

## 11.8.A.2

As a result of conservation of charge, each of the capacitors in series must have the same magnitude of charge on each plate.

## SUGGESTED SKILLS

## 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

## 3.A

Create experimental procedures that are appropriate for a given scientific question.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

continued on next page

## LEARNING OBJECTIVE

**11.8.B**

Describe the behavior of a circuit containing combinations of resistors and capacitors.

## ESSENTIAL KNOWLEDGE

**11.8.B.1**

The charge on a capacitor or the current in a resistor in an RC circuit can be described by a fundamental differential equation derived from Kirchhoff's loop rule.

*Derived equation:*

$$\mathcal{E} = \frac{dq}{dt}R + \frac{q}{C}$$

**11.8.B.2**

The time constant ( $\tau$ ) is a significant feature of an RC circuit.

**11.8.B.2.i**

The time constant of an RC circuit is a measure of how quickly the capacitor will charge or discharge and is defined as

$$\tau = R_{\text{eq}}C_{\text{eq}}.$$

**11.8.B.2.ii**

For a charging capacitor, the time constant represents the time required for the capacitor's charge to increase from zero to approximately 63 percent of its final asymptotic value.

**11.8.B.2.iii**

For a discharging capacitor, the time constant represents the time required for the capacitor's charge to decrease from fully charged to approximately 37 percent of its initial value.

**11.8.B.3**

The potential difference across a capacitor and the current in the branch of the circuit containing the capacitor each change over time as the capacitor charges and discharges, but both will reach a steady state after a long time interval.

**11.8.B.3.i**

Immediately after being placed in a circuit, an uncharged capacitor acts like a wire, and charge can easily flow to or from the plates of the capacitor.

*continued on next page*

**LEARNING OBJECTIVE****11.8.B**

Describe the behavior of a circuit containing combinations of resistors and capacitors.

**ESSENTIAL KNOWLEDGE****11.8.B.3.ii**

As a capacitor charges, changes to the potential difference across the capacitor affect the charge on the plates of the capacitor, the current in the circuit branch in which the capacitor is located, and the electric potential energy stored in the capacitor.

**11.8.B.3.iii**

The potential difference across a capacitor, the current in the circuit branch in which the capacitor is located, and the electric potential energy stored in the capacitor all change with respect to time and asymptotically approach steady state conditions.

**11.8.B.3.iv**

After a long time, a charging capacitor approaches a state of being fully charged, reaching a maximum potential difference at which there is zero current in the circuit branch in which the capacitor is located.

**11.8.B.3.v**

Immediately after a charged capacitor begins discharging, the amount of charge on the capacitor and the energy stored in the capacitor begin to decrease.

**11.8.B.3.vi**

As a capacitor discharges, the amount of charge on the capacitor, the potential difference across the capacitor, and the current in the circuit branch in which the capacitor is located all decrease until a steady state is reached.

**11.8.B.3.vii**

After either charging or discharging for times much greater than the time constant, the capacitor and the relevant circuit branch may be modeled using steady-state conditions.



THIS PAGE IS INTENTIONALLY LEFT BLANK.

**AP PHYSICS C: ELECTRICITY  
AND MAGNETISM**

**UNIT 12**

**Magnetic  
Fields and  
Electro-  
magnetism**



**10–20%**  
AP EXAM WEIGHTING



**~10/~20**  
CLASS PERIODS

---

The icon consists of a white circle containing a blue square with the letters 'AP' in white. Below the square is a small black monitor icon with two lines representing a screen.

Remember to go to [AP Classroom](#) to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

### Progress Check 12

**Multiple-Choice: ~18 questions**

**Free-Response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# Magnetic Fields and Electromagnetism



## Developing Understanding

### ESSENTIAL QUESTIONS

- Why are large-scale, charged-particle accelerators, such as those at CERN, in the shape of a circle?
- How does a guitar pickup work?
- Why does an electric current deflect the needle of a compass?
- Why does the deflection of a pair of parallel conducting wires depend on the direction of current in the wires?

In previous units, students discovered that the electric field allows charged objects to interact without contact. Unit 12 introduces students to magnetism and how magnetic fields are generated, behave, and relate to electricity. Students will learn how magnetic fields impact motion and interact with other magnetic fields. Laboratory investigations and/or activities should be provided for students to apply both the Biot–Savart Law (using calculations to determine the strength of a magnetic field) and Ampère’s Law (deriving mathematical relationships which relate the magnitude of the magnetic field to current). This knowledge from previous units helps students to make connections between electric fields and magnetic fields as well as between Gauss’s Law and Ampère’s Law.

## Building the Science Practices

2.C 2.D 3.B 3.C

Unit 12 provides opportunities for students to compare physical quantities between scenarios or at different times in a single scenario (2.C), as well as determine new values of quantities using functional dependencies between variables (2.D). From there, students can also make and justify claims based on these physical principles and functional relationships (3.B, 3.C).

For example, students could describe conceptually what happens to the net force exerted on an electron moving through a magnetic field if the speed of the electron increases, and then justify what impact that change will have on the radius of the path of the electron. By the end of the unit, it is important for students to be comfortable with making claims about the reasonableness of their claims and justifications made with functional dependence (2.D, 3.C), starting with the first principles of physics.

## Preparing for the AP Exam

On both the multiple-choice and free-response sections of the AP Physics C: Electricity and Magnetism Exam, students need to be able to describe the relationships between physical quantities in order to articulate the effects of changing a physical quantity in a scenario. Therefore, students will benefit from opportunities to investigate changes in systems, including practicing using fundamental principles of physics to decide whether a quantity will increase, decrease, or remain the same when another quantity is changed. Additionally, when writing justifications for claims, simply referencing an equation, law or physical principle is not sufficient. For example, stating that the induced current in a loop is clockwise because of “Lenz’s Law” is not complete enough of an answer to earn credit on the free-response section of the exam. Instead, students should cultivate the habit of including references to the right hand rule and the resistance to the change in magnetic flux in their responses.

## UNIT AT A GLANCE

Topics	Suggested Skills
<b>12.1 Magnetic Fields</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li><li><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li></ul>
<b>12.2 Magnetism and Moving Charges</b>	<ul style="list-style-type: none"><li><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>12.3 Magnetic Fields of Current-Carrying Wires and the Biot–Savart Law</b>	<ul style="list-style-type: none"><li><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</li><li><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li></ul>
<b>12.4 Ampère’s Law</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 12.  
Review the results in class to identify and address any student misunderstandings.

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	12.2	<b>Sharing and Responding</b> Have students research some model of old CRT television. Ask them to find the potential difference through which electrons are accelerated to determine their speed entering the magnetic field region, then the strength of the magnetic field needed to deflect them to different points on the screen.
2	12.3	<b>Sharing and Responding</b> Have students research the current in an east-west traveling high-tension power line and find the length of such a power line near their home. Also have them research the magnetic field strength of Earth and determine the force Earth's magnetic field exerts on the wire. Then, have students compare the force on the wire from Earth's magnetic field to the wire's weight (which is the force on the wire from Earth's gravitational field).
3	12.4	<b>Create a Plan</b> Have students use their research of high-tension power lines to determine how far from such a power line a person must stand before the magnetic field created by the power line is equal to that of Earth. Have them conduct the same research for the distance from a wire to a hair dryer or vacuum cleaner in the home.
4	12.4	<b>Create a Plan</b> Have students research the magnetic field strength of a typical MRI machine, as well as the magnetic field strength of the machine's length and radius. Next, have them determine how that magnetic field can be created with a same-radius, same-length solenoid. Include factors such as length and radius of the wire making the solenoid, its resistance, the current and voltage necessary, and the cost of the wire.
5	12.4	<b>Desktop Experiment Tasks</b> Provide students with a piece of rectangular aluminum foil about 1 foot wide and 2 feet long. Connect it to a battery and resistance (so current travels the long way) and orient it so that current travels toward magnetic north. A compass on the foil should deflect between 30 and 60 degrees (or adjust the voltage/resistance). Then, have students use the compass deflection and Ampère's Law to determine the magnetic field of Earth.

## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## TOPIC 12.1

## Magnetic Fields

## Required Course Content

## LEARNING OBJECTIVE

## 12.1.A

Describe the properties of a magnetic field.

## ESSENTIAL KNOWLEDGE

## 12.1.A.1

A magnetic field is a vector field that can be used to determine the magnetic force exerted on moving electric charges, electric currents, or magnetic materials.

## 12.1.A.1.i

Magnetic fields can be produced by magnetic dipoles or combinations of dipoles, but never by monopoles.

## 12.1.A.1.ii

Magnetic dipoles have north and south polarity.

## 12.1.A.2

A magnetic field is a vector quantity and can be represented using vector field maps.

## 12.1.A.3

Magnetic field lines must form closed loops, as described by Gauss's law for magnetism.

## 12.1.A.3.i

Maxwell's equations are the collection of equations that fully describe electromagnetism. Gauss's law for magnetism is Maxwell's second equation.

*Relevant equation:*

$$\oint \vec{B} \cdot d\vec{A} = 0$$

## 12.1.A.3.ii

Magnetic fields in a bar magnet form closed loops, with the external magnetic field pointing away from one end (defined as the north pole) and returning to the other end (defined as the south pole).

**LEARNING OBJECTIVE****12.1.B**

Describe the magnetic behavior of a material as a result of the configuration of magnetic dipoles in the material.

**ESSENTIAL KNOWLEDGE****12.1.B.1**

Magnetic dipoles result from the circular or rotational motion of electric charges. In magnetic materials, this can be the motion of electrons.

**12.1.B.1.i**

Permanent magnetism and induced magnetism are system properties that both result from the alignment of magnetic dipoles within a system.

**12.1.B.1.ii**

No magnetic north pole is ever found in isolation from a south pole. For example, if a bar magnet is broken in half, both halves are magnetic dipoles.

**12.1.B.1.iii**

Magnetic poles of the same polarity will repel; magnetic poles of opposite polarity will attract.

**12.1.B.1.iv**

The magnitude of the magnetic field from a magnetic dipole decreases with increasing distance from the dipole.

**12.1.B.2**

A magnetic dipole, such as a magnetic compass, placed in a magnetic field will tend to align with the magnetic field.

**12.1.B.3**

A material's composition influences its magnetic behavior in the presence of an external magnetic field.

**12.1.B.3.i**

Ferromagnetic materials such as iron, nickel, and cobalt can be permanently magnetized by an external field that causes the alignment of magnetic domains or atomic magnetic dipoles.

**12.1.B.3.ii**

Paramagnetic materials such as aluminum, titanium, and magnesium interact weakly with an external magnetic field, in that the magnetic dipoles of the material do not remain aligned after the external field is removed.

*continued on next page*



## LEARNING OBJECTIVE

**12.1.B**

Describe the magnetic behavior of a material as a result of the configuration of magnetic dipoles in the material.

**12.1.C**

Describe the magnetic permeability of a material.

## ESSENTIAL KNOWLEDGE

**12.1.B.3.iii**

All materials have the property of diamagnetism, in that their electronic structure creates a usually weak alignment of the dipole moments of the material opposite the external magnetic field.

**12.1.B.4**

Earth's magnetic field may be approximated as a magnetic dipole.

**12.1.C.1**

Magnetic permeability is a measurement of the amount of magnetization in a material in response to an external magnetic field.

**12.1.C.2**

Free space has a constant value of magnetic permeability, known as the vacuum permeability  $\mu_0$ , that appears in equations representing physical relationships.

**12.1.C.3**

The permeability of matter has values different from that of free space and arises from the matter's composition and arrangement. It is not a constant for a material and varies based on many factors, including temperature, orientation, and strength of the external field.

## TOPIC 12.2

## Magnetism and Moving Charges

## Required Course Content

## LEARNING OBJECTIVE

## 12.2.A

Describe the magnetic field produced by moving charged objects.

## 12.2.B

Describe the force exerted on moving charged objects by a magnetic field.

## ESSENTIAL KNOWLEDGE

## 12.2.A.1

A single moving charged object produces a magnetic field.

## 12.2.A.1.i

The magnetic field at a particular point produced by a moving charged object depends on the object's velocity and the distance between the point and the object.

## 12.2.A.1.ii

At a point in space, the direction of the magnetic field produced by a moving charged object is perpendicular to both the velocity of the object and the position vector from the object to that point in space and can be determined using the right-hand rule.

## 12.2.A.1.iii

The magnitude of the magnetic field is a maximum when the velocity vector and the position vector from the object to that point in space are perpendicular.

## 12.2.B.1

A magnetic field will exert a force on a charged object moving within that field, with magnitude and direction that depend on the cross-product of the charge's velocity and the magnetic field.

Relevant equation:

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

## SUGGESTED SKILLS

## 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.A

Create experimental procedures that are appropriate for a given scientific question.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

continued on next page

## LEARNING OBJECTIVE

**12.2.B**

Describe the force exerted on moving charged objects by a magnetic field.

## ESSENTIAL KNOWLEDGE

**12.2.B.2**

In a region containing both a magnetic field and an electric field, a moving charged object will experience independent forces from each field.

**12.2.B.3**

The Hall effect describes the potential difference created in a conductor by an external magnetic field that has a component perpendicular to the direction of charges moving in the conductor.

## TOPIC 12.3

# Magnetic Fields of Current-Carrying Wires and the Biot-Savart Law

## Required Course Content

### LEARNING OBJECTIVE

**12.3.A**

Describe the magnetic field produced by a current-carrying wire.

### ESSENTIAL KNOWLEDGE

**12.3.A.1**

The Biot-Savart law defines the magnitude and direction of a magnetic field created by an electrical current.

*Relevant equation:*

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{\ell} \times \hat{r})}{r^2}$$

**12.3.A.2**

The magnetic field vectors around a small segment of a current-carrying wire are tangent to concentric circles centered on that wire. The field has no component toward, away from, or parallel to the segment of the current-carrying wire.

**12.3.A.3**

The Biot-Savart law can be used to derive the magnitudes and directions of magnetic fields around segments of current-carrying wires, for example at the center of a circular loop of wire.

*Derived equation:*

$$B_{\text{center of loop}} = \frac{\mu_0 I}{2R}$$

### SUGGESTED SKILLS

**1.C**

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

**2.A**

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

**2.D**

Predict new values or factors of change of physical quantities using functional dependence between variables.

**3.C**

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

*continued on next page*

**LEARNING OBJECTIVE****12.3.B**

Describe the force exerted on current-carrying wires by a magnetic field.

**ESSENTIAL KNOWLEDGE****12.3.B.1**

A magnetic field will exert a force on a current-carrying wire.

Relevant equation:

$$\vec{F}_B = \int I(d\vec{\ell} \times \vec{B})$$

**BOUNDARY STATEMENT**

*AP Physics C: Electricity & Magnetism only expects students to perform quantitative analysis of certain cases of current-carrying conductors using the Biot-Savart law, such as at a location along the perpendicular bisector of a straight conductor, at a location along the central axis of a circular loop, or at the center of a segment of a circular loop.*

## TOPIC 12.4

## Ampère's Law

## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## Required Course Content

## LEARNING OBJECTIVE

## 12.4.A

Use Ampère's law to describe the magnetic field created by a moving charge carrier.

## ESSENTIAL KNOWLEDGE

## 12.4.A.1

Ampère's law relates the magnitude of the magnetic field to the current enclosed by a closed imaginary path called an Amperian loop.

*Relevant equation:*

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

## 12.4.A.1.i

Ampère's law can be used to determine the magnetic field near a long, straight current-carrying wire.

*Derived equation:*

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$

## 12.4.A.1.ii

Unless otherwise stated, all solenoids are assumed to be very long, with uniform magnetic fields inside the solenoids and negligible magnetic fields outside the solenoids.

## 12.4.A.1.iii

Ampère's law can be used to determine the magnetic field inside of a long solenoid.

*Derived equation:*

$$B_{\text{sol}} = \mu_0 nI$$

## 12.4.A.2

An Amperian loop is a closed path around a current-carrying conductor.

*continued on next page*

## LEARNING OBJECTIVE

## 12.4.A

Use Ampère's law to describe the magnetic field created by a moving charge carrier.

## ESSENTIAL KNOWLEDGE

## 12.4.A.3

The principle of superposition can be used to determine the net magnetic field at a point in space created by various combinations of current-carrying conductors, or conducting loops, segments, or cylinders.

## 12.4.A.4

Maxwell's equations are the collection of equations that fully describe electromagnetism. Maxwell's fourth equation is Ampère's law with Maxwell's addition; it states that magnetic fields can be generated by electric current (Ampère's law) and that a changing electric field creates a magnetic field, similar to the way a moving charge creates a magnetic field (Maxwell's addition).

Relevant equations:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

## BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism only expects quantitative application of Ampère's law limited to situations involving symmetrical magnetic fields. Long straight wires, long solenoids carrying currents, as well as conductive slabs or cylindrical conductors carrying a current density, are the types of shapes to which Ampère's law will be applied on the AP Physics C: Electricity & Magnetism Exam.*

## BOUNDARY STATEMENT

*AP Physics C: Electricity & Magnetism does not expect students to use Maxwell's fourth equation with a changing electric field. However, students should understand that a changing electric field generates a magnetic field.*

**AP PHYSICS C: ELECTRICITY  
AND MAGNETISM**

**UNIT 13**

**Electro-  
magnetic  
Induction**



**10–20%**  
AP EXAM WEIGHTING



**~10/~20**  
CLASS PERIODS

---



The icon consists of a white circle containing a blue square with the letters 'AP' in white. Below the square is a small blue monitor icon with two lines representing a screen.

Remember to go to [AP Classroom](#) to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topic and skills.

### Progress Check 13

**Multiple-Choice: ~18 questions**

**Free-Response: 4 questions**

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

# Electromagnetic Induction



## Developing Understanding

### ESSENTIAL QUESTIONS

- How does an electric motor work?
- How does pushing the doorbell produce a sound inside the house?
- How does an antenna work?
- How are sound waves generated by your headphones or speakers from a digital recording of a song?
- How does a Wi-Fi internet connection work?

Throughout the course, students explored the vital roles electricity and magnetism play in our daily lives. Unit 13 examines electromagnetism through the concept of electromagnetic induction and the application of Maxwell's equations. Through classroom discussions and problem-solving, students will study, apply, and analyze the concept of induction, as well as investigate the relationship between Faraday's Law and Lenz's Law. Additionally, students are expected to call upon their knowledge obtained in earlier units—particularly that of charges, currents, and electric and magnetic fields—to better understand Maxwell's equations and to be able to mathematically demonstrate, as well as reason with, how these fields are generated.

## Building the Science Practices

2.C 2.D 3.B 3.C

In Unit 13, students are expected to use representations, as well as data and/or fundamental laws of physics, to make and provide reasoning to justify, a claim (**3.B, 3.C**) by using functional dependence and the relationships between variables. (**2.C, 2.D**). By the end of Unit 13, students should be proficient in calculating unknown quantities with unit and/or symbolic expressions from known quantities by selecting and following a logical computational pathway. However, simply being able to solve for a final answer is insufficient; students may benefit from practice crafting clear, concise, arguments, derivations, and calculations that follow a logical pathway. For example, describing how a solenoid resists a change in electrical current due the required change in magnetic field within that solenoid is a valuable skill, in addition to deriving a mathematical equation that shows the current in the solenoid as a function of time.

## Preparing for the AP Exam

In both the Multiple-Choice and Free Response sections of the AP Physics C: Electricity and Magnetism Exam, students will be asked to identify and analyze functional relationships. Students may be asked to determine the relationships between variables and models to describe a physical situation. For example, students may be asked to determine the changes in an induced current when the magnetic flux through the loop is changing or if the magnetic field is turned on and off. Direct instruction supporting claims based on evidence obtained through an application of functional dependence will help students be more successful on the AP Physics C: Electricity and Magnetism Exam. Laboratory investigations about the behavior of circuits when a solenoid is in the circuit are valuable exercises that concretely demonstrate principles and ideas that are often abstract.

## UNIT AT A GLANCE

Topics	Suggested Skills
<b>13.1 Magnetic Flux</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>13.2 Electromagnetic Induction</b>	<ul style="list-style-type: none"><li><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</li><li><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</li></ul>
<b>13.3 Induced Currents and Magnetic Forces</b>	<ul style="list-style-type: none"><li><b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.</li><li><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</li><li><b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>
<b>13.4 Inductance</b>	<ul style="list-style-type: none"><li><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</li><li><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</li><li><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</li><li><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</li></ul>

*continued on next page*

**UNIT AT A GLANCE** *(cont'd)*

Topics	Suggested Skills
<b>13.5 Circuits with Resistors and Inductors (LR Circuits)</b>	<p><b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</p> <p><b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</p>
<b>13.6 Circuits with Capacitors and Inductors (LC Circuits)</b>	<p><b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.</p> <p><b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities by selecting and following a logical computational pathway.</p> <p><b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.</p> <p><b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.</p> <p><b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</p>



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 13.  
Review the results in class to identify and address any student misunderstandings.

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 125 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	13.2	<b>Desktop Experiment Tasks</b> Have students spin a magnet within a coil of wire and measure the spin rate, area, number of coils, and peak voltage induced. From this, have students estimate the strength of the magnetic field generated by the magnet.
2	13.2	<b>Qualitative Reasoning</b> Have students describe qualitatively how electromagnetic braking works, including how electromagnetic brakes are structured, how e-brakes can recharge a battery, and how e-brakes can double as electric motors.
3	13.2	<b>Construct an Argument</b> Have students use Maxwell's equations to construct arguments for the following: Why there can be no magnetic monopoles, what eddy currents are and why they exist, why a surface entirely within conducting material must have zero net charge within, and why there must be electric currents within Earth's core.
4	13.5	<b>Desktop Experiment Tasks</b> Have students construct their own solenoid (or provide one) and measure its inductance using an RL circuit (measure the time constant and the R to get L). Then, have them repeat the experiment with iron or steel in the core of the inductor to get the increased L.
5	13.5	<b>Graph and Switch</b> Break students into groups of 2. Have Student A construct quantitative graphs of current versus time and voltage versus time for inductor, resistor 1, and resistor 2 that shows current/voltage before and after a switch opens/closes. Student B must then construct the circuit with the switch, R1, R2, L.

## TOPIC 13.1

# Magnetic Flux

### Required Course Content

#### LEARNING OBJECTIVE

##### 13.1.A

Describe the magnetic flux through an arbitrary area or geometric shape.

#### ESSENTIAL KNOWLEDGE

##### 13.1.A.1

For a magnetic field  $\vec{B}$  that is constant across an area  $\vec{A}$ , the magnetic flux through the area is defined as

$$\Phi_B = \vec{B} \cdot \vec{A}.$$

##### 13.1.A.1.i

The area vector is defined as perpendicular to the plane of the surface and outward from a closed surface.

##### 13.1.A.1.ii

The sign of flux is given by the dot product of the magnetic field vector and the area vector.

##### 13.1.A.2

The total magnetic flux passing through a surface is defined by the surface integral of the magnetic field over the surface area.

*Relevant equation:*

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

#### SUGGESTED SKILLS

##### 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

##### 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

##### 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

##### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## SUGGESTED SKILLS

## 1.B

Create quantitative graphs with appropriate scales and units, including plotting data.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.A

Create experimental procedures that are appropriate for a given scientific question.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

## TOPIC 13.2

# Electromagnetic Induction

## Required Course Content

### LEARNING OBJECTIVE

## 13.2.A

Describe the induced electric potential difference resulting from a change in magnetic flux.

### ESSENTIAL KNOWLEDGE

## 13.2.A.1

Faraday's law describes the relationship between changing magnetic flux and the resulting induced emf in a system.

*Relevant equation:*

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(\vec{B} \cdot \vec{A})}{dt}$$

## 13.2.A.1.i

When the area of the surface being considered is constant, the induced emf is equal to the area multiplied by the rate of change in the component of the magnetic field perpendicular to the surface.

## 13.2.A.1.ii

When the magnetic field is constant, the induced emf is equal to the magnetic field multiplied by the rate of change in area perpendicular to the magnetic field.

## 13.2.A.1.iii

When an emf is induced in a long solenoid, the total induced emf is equal to the induced emf in a single loop multiplied by the number of loops in the solenoid.

*Relevant equation:*

$$|\mathcal{E}_{\text{sol}}| = N \left| \frac{d\Phi_B}{dt} \right|$$

*continued on next page*

**LEARNING OBJECTIVE****13.2.A**

Describe the induced electric potential difference resulting from a change in magnetic flux.

**ESSENTIAL KNOWLEDGE****13.2.A.2**

Lenz's law is used to determine the direction of an induced emf resulting from a changing magnetic flux.

**13.2.A.2.i**

An induced emf generates a current that creates a magnetic field that opposes the change in magnetic flux.

**13.2.A.2.ii**

The right-hand rule is used to determine the relationships between current, emf, and magnetic flux.

**13.2.A.3**

Maxwell's equations are the collection of equations that fully describe electromagnetism. Maxwell's third equation is Faraday's law of induction, which describes the relationship between a changing magnetic flux and an induced electric field.

*Relevant equation:*

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

**13.2.A.4**

Maxwell's equations can be used to show that electric and magnetic fields obey wave equations and that electromagnetic waves travel at a constant speed in free space.

*Derived equation:*

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

**BOUNDARY STATEMENT**

*AP Physics C: Electricity & Magnetism does not expect students to mathematically derive the speed of light in free space from Maxwell's equations. This relationship is included above solely as an indication of the further applications, implications, and connections to physical phenomena that students may study in more advanced physics courses.*



## SUGGESTED SKILLS

## 1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

## 2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

## 2.D

Predict new values or factors of change of physical quantities using functional dependence between variables.

## 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## TOPIC 13.3

# Induced Currents and Magnetic Forces

## Required Course Content

### LEARNING OBJECTIVE

## 13.3.A

Describe the force exerted on a conductor due to the interaction between an external magnetic field and an induced current within that conductor.

### ESSENTIAL KNOWLEDGE

## 13.3.A.1

When an induced current is created in a conductive loop, the already-present magnetic field will exert a magnetic force on the moving charge carriers within the loop.

*Relevant equation:*

$$\vec{F}_B = \int I(d\vec{\ell} \times \vec{B})$$

## 13.3.A.2

When current is induced in a conducting loop, magnetic forces are only exerted on the segments of the loop that are within the external magnetic field. These magnetic forces may cause translational or rotational acceleration.

## 13.3.A.3

The force on a conducting loop is proportional to the induced current in the loop, which depends on the rate of change of magnetic flux, the resistance of the loop, and the velocity of the loop.

## 13.3.A.4

Newton's second law can be applied to a conducting loop moving in a magnetic field as it experiences an induced emf.

## TOPIC 13.4

# Inductance

### SUGGESTED SKILLS

#### 1.C

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

#### 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

#### 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

#### 3.B

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

## Required Course Content

### LEARNING OBJECTIVE

#### 13.4.A

Describe the physical and electrical properties of an inductor.

### ESSENTIAL KNOWLEDGE

#### 13.4.A.1

Inductance is the tendency of a conductor to oppose a change in electrical current.

##### 13.4.A.1.i

Inductance of a conductor depends on the physical properties of the conductor. Straight wires are typically modeled as having zero inductance.

##### 13.4.A.1.ii

An inductor, such as a solenoid, is a circuit element that has significant inductance.

##### 13.4.A.1.iii

The inductance of a solenoid is dependent on the total number of turns, the length of the solenoid, the cross-sectional area of the solenoid, and magnetic permeability of the solenoid's core.

*Relevant equation:*

$$L_{\text{sol}} = \frac{\mu_{\text{core}} N^2 A}{\ell}$$

#### 13.4.A.2

Inductors store energy in the magnetic field that is generated by current in the inductor.

*Relevant equation:*

$$U_L = \frac{1}{2} LI^2$$

##### 13.4.A.2.i

The energy stored in the magnetic field generated by an inductor in which current is flowing can be dissipated through a resistor or used to charge a capacitor.

*continued on next page*

## LEARNING OBJECTIVE

## 13.4.A

Describe the physical and electrical properties of an inductor.

## ESSENTIAL KNOWLEDGE

## 13.4.A.2.ii

The transfer of energy generated in an inductor to other forms of energy obeys conservation laws.

## 13.4.A.3

By applying Faraday's law to an inductor and using the definition of inductance, induced emf can be related to inductance and the rate of change of current.

*Relevant equation:*

$$\mathcal{E}_i = -L \frac{dI}{dt}$$

## TOPIC 13.5

Circuits with Resistors  
and Inductors  
(LR Circuits)

## Required Course Content

## LEARNING OBJECTIVE

## 13.5.A

Describe the physical and electrical properties of a circuit containing a combination of resistors and a single inductor.

## ESSENTIAL KNOWLEDGE

## 13.5.A.1

A resistor will dissipate energy that was stored in an inductor as the current changes.

## 13.5.A.2

Kirchhoff's loop rule can be applied to a series LR circuit with a battery of emf  $\mathcal{E}$ , resulting in a differential equation that describes the current in the loop.

*Derived equation:*

$$\mathcal{E} = IR + L \frac{dI}{dt}$$

## 13.5.A.3

The time constant is a significant feature of the behavior of an LR circuit.

## 13.5.A.3.i

The time constant of a circuit is a measure of how quickly an LR circuit will reach a steady state and is described with the equation

$$\tau = \frac{L}{R_{\text{eq}}}$$

## 13.5.A.3.ii

The time constant represents the time an LR circuit would take to reach a steady state if the system continued to change at the initial rate of change.

## 13.5.A.3.iii

For an inductor that has zero initial current, the time constant represents the time required for the current in the inductor to reach approximately 63 percent of its final asymptotic value.

## SUGGESTED SKILLS

## 1.C

Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

## 2.A

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

## 2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

## 3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

*continued on next page*

## LEARNING OBJECTIVE

**13.5.A**

Describe the physical and electrical properties of a circuit containing a combination of resistors and a single inductor.

## ESSENTIAL KNOWLEDGE

**13.5.A.3.iv**

For an inductor with an initial current, the time constant represents the time required for the current in the inductor to reach approximately 37 percent of its initial value.

**13.5.A.4**

The electric properties of inductors change during the time interval in which the current in the inductor changes, but will exhibit steady state behavior after a long time interval.

**13.5.A.4.i**

When a switch is initially closed or opened in a circuit containing an inductor, the induced emf will be equal in magnitude and opposite in direction to the applied potential difference across the branch containing the inductor.

**13.5.A.4.ii**

The potential difference across an inductor, the current in the inductor, and the energy stored in the inductor are exponential with respect to time and have asymptotes that are determined by the initial conditions of the circuit.

**13.5.A.4.iii**

After a time much greater than the time constant of the circuit, an inductor will behave as a conducting wire with zero resistance.

## TOPIC 13.6

# Circuits with Capacitors and Inductors (LC Circuits)

## Required Course Content

### LEARNING OBJECTIVE

**13.6.A**

Describe the physical and electrical properties of a circuit containing a combination of capacitors and a single inductor.

### ESSENTIAL KNOWLEDGE

**13.6.A.1**

In circuits containing only a charged capacitor and an inductor (LC circuits), the maximum current in the inductor can be determined using conservation of energy within the circuit.

**13.6.A.2**

In LC circuits, the time dependence of the charge stored in the capacitor can be modeled as simple harmonic motion.

*Derived equation:*

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

**13.6.A.3**

The angular frequency of an oscillating LC circuit can be derived from the differential equation that describes an LC circuit.

*Derived equation:*

$$\omega = \frac{1}{\sqrt{LC}}$$

### SUGGESTED SKILLS

**1.B**

Create quantitative graphs with appropriate scales and units, including plotting data.

**2.B**

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

**2.C**

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

**3.A**

Create experimental procedures that are appropriate for a given scientific question.

**3.B**

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

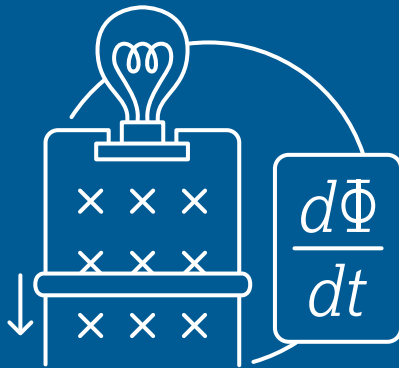
THIS PAGE IS INTENTIONALLY LEFT BLANK.

**AP PHYSICS C: ELECTRICITY AND MAGNETISM**

---

# Laboratory Investigations





# Lab Experiments

---

Although laboratory work has often been separated from classroom work, experience and experiment are often more instructionally effective when flexibly integrated into the development of concepts. When students build their own conceptual understanding of the principles of physics, their familiarity with the concrete evidence for their ideas leads to deeper understanding of course concepts and gives them a sense of ownership of the knowledge they have constructed.

Scientific inquiry experiences in AP Physics C: Electricity and Magnetism should be designed and implemented with increasing student involvement to help enhance inquiry learning and the development of critical thinking and problem-solving skills and abilities. Typically, the level of investigations in an AP Physics C: Electricity and Magnetism classroom should focus primarily on the continuum between guided and open inquiry. However, depending on students' familiarity with a topic, a given laboratory experience might incorporate a sequence involving all four levels of inquiry (confirmation, structured inquiry, guided inquiry, and open inquiry).

## Lab Manuals and Lab Notebooks

Many publishers and science classroom material distributors offer affordable lab manuals with outlined experiments and activities as well as lab notebooks for recording lab data and observations. Students can use any type of notebook to fulfill the lab notebook requirement, even an online document. Consider the needs of the classroom when deciding what type of lab notebook to use.

## Lab Materials

A wide range of equipment may be used in the physics laboratory, from generic lab items, such as metersticks, rubber balls, springs, string, metal spheres, calibrated mass sets, beakers, glass and cardboard tubes, electronic balances, stopwatches, clamps, and ring stands, to items more specific to physics, such as tracks, carts, light bulbs, resistors, magnets, and batteries. Successful guided inquiry student work can be accomplished with both simple, inexpensive materials and with more sophisticated

physics equipment, such as air tracks, force sensors, and oscilloscopes. Remember that the AP Physics C: Electricity and Magnetism lab should provide an experience for students equivalent to that of a college laboratory, so teachers are encouraged to make every effort to provide a range of experiences—from experiments students contrive from plumbing pipe, string, and duct tape, to experiments in which students gather and analyze data using calculators or computer-interfaced equipment.

There are avenues that teachers can explore as a means of getting access to more expensive equipment, such as computers and probes. Probes can often be rented for short periods of time from instrument suppliers. Alternatively, local colleges or universities may allow high school students to complete a lab as a field trip on their campus, or they may allow teachers to borrow their equipment. They may even donate their old equipment. Some schools have partnerships with local businesses that can help with laboratory equipment and materials. Teachers can also utilize online donation sites such as Donors Choose and Adopt-A-Classroom.

## Lab Time

For AP Physics C: Electricity and Magnetism to be comparable to a college physics course, it is critical that teachers make laboratory work an important part of their curriculum. An analysis of data from AP Physics C: Electricity and Magnetism examinees, regarding the length of time they spent per week in the laboratory, shows that increased laboratory time correlates with higher AP scores. Flexible or modular scheduling must be implemented to meet the time requirements identified in the course outline. Furthermore, it is important that the AP Physics laboratory program be adapted to local conditions and funding as it aims to offer the students a well-rounded experience with experimental physics. Adequate laboratory facilities should be provided so that each student has a work space where equipment and materials can be left overnight if necessary. Sufficient laboratory equipment for the anticipated enrollment and appropriate instruments should be provided. Students in AP Physics should have access to computers with software appropriate for processing laboratory data and writing reports.

# How to Set Up a Lab Program

---

Physics is a way of approaching scientific discovery that requires personal observation and physical experimentation. Being successful in this endeavor requires students to synthesize and use a broad spectrum of knowledge and skills, including mathematical, computational, experimental, and practical skills, and to develop habits of mind that might be characterized as thinking like a physicist. Student-directed, inquiry-based lab experience supports the AP Physics C: Electricity and Magnetism course and AP Course Audit curricular requirements. It provides opportunities for students to design experiments, collect data, apply mathematical routines and methods, and refine testable explanations and predictions. The AP Physics C: Electricity and Magnetism course should include a hands-on laboratory component comparable to a semester-long introductory college-level physics laboratory. Students must spend a minimum of 25 percent of instructional time engaged in hands-on laboratory work.

The AP Physics C: Electricity and Magnetism Exam directly assesses the learning objectives of the course framework, which means that the inclusion of appropriate experiments aligned with those learning objectives is important for student success. Teachers should select experiments that provide students with the broadest laboratory experience possible. We encourage teachers to be creative in designing their lab program while ensuring that students explore and develop understandings of the core techniques of writing experimental procedures and analyzing data. After completion, students should be able to describe how to construct knowledge, model (create an abstract representation of a real system), design experiments, analyze visual data, and communicate physics. Students should also develop an understanding of how changes in the design of the experiments would affect the outcome of their results. Many questions on the AP Exam are written in an experimental context, so these skills will prove invaluable for both concept comprehension and exam performance. Because AP Physics C: Electricity and Magnetism is equivalent

to a college course, the equipment and time allotted to laboratories should be similar to that in a college course. Therefore, schools must ensure that students have access to scientific equipment and all materials necessary to conduct hands-on, college-level physics laboratory investigations.

## Getting Students Started

There are no prescriptive “steps” to the iterative process of inquiry-based investigations. However, there are some common characteristics of inquiry that will support students in designing their investigations. Often, this simply begins with using the learning objectives to craft a question for students to investigate. Teachers may choose to give students a list of materials they are allowed to use in their design or require that students request the equipment they feel they need to investigate the question. Working with learning objectives to craft questions may include the following:

- Selecting learning objectives from the course framework that relate to the subject under study and that may set forth specific tasks, in the form of “Design an experiment to ...”
- Rephrasing or refining the learning objectives that align to the unit of study to create an inquiry-based investigation for students.

Students should be given latitude to make design modifications or ask for additional equipment appropriate for their design. It is also helpful for individual groups to report to the class their basic design to elicit feedback on feasibility. Guided student groups can proceed through the experiment with the teacher allowing them the freedom to make mistakes—as long as those mistakes don’t endanger students or equipment, or lead the groups too far off task. Students should have many opportunities for post-lab reporting so that groups can understand the successes and challenges of individual lab designs.

## Communication, Group Collaboration, and the Laboratory Record

Laboratory work is an excellent means through which students can develop and practice communication skills. Success in subsequent work in physics depends heavily on an ability to communicate observations, ideas, and conclusions to others. By working together in a truly collaborative manner to plan and carry out experiments, students learn oral communication skills and teamwork. Students must be encouraged to take full individual responsibility for the success, or failure, of the collaboration. After students are given a question for investigation, they may present their findings in either a written or an oral report to the teacher and class for feedback and critique on their final design and results. Students should be encouraged to critique and challenge one another's claims based on the evidence collected during the investigation.

## Laboratory Safety

Giving students the responsibility for designing their own laboratory experience involves special responsibilities for teachers. To ensure a safe working environment, teachers should first provide the limitations and safety precautions necessary for potential procedures and equipment students may use during their investigation. Teachers should also provide specific guidelines prior to students' discussion on investigation designs for each experiment so that those precautions can be incorporated into final student-selected lab designs and included in the background or design plan in a laboratory record. It may also be

helpful to print the precautions that apply to that specific lab as safety notes to place on the desk or wall near student workstations. Additionally, a general set of safety guidelines should be set forth for students at the beginning of the course. The following is a list of possible general guidelines teachers may post.

- Before each lab, make sure you know and record the potential hazards involved in the investigation, as well as the precautions you will take to stay safe.
- Before using equipment, make sure you know the proper method of use to acquire good data and avoid damage to equipment.
- Know where safety equipment is located in the lab, such as the fire extinguisher, safety goggles, and the first aid kit.
- Follow the teacher's special safety guidelines as set forth prior to each experiment. (Students should record these as part of their design plan for a lab.)
- When in doubt about the safety or advisability of a procedure, check with the teacher before proceeding.

Teachers should interact constantly with students as they work to observe safety practices and anticipate and discuss with them any problems that may arise. Walking among student groups and, asking questions allows, teachers to keep the pulse of what students are doing and maintain a watchful eye for potential safety issues.

# Laboratory Investigations

---

## Introduction

Laboratory investigations, experiments, and activities (also called “labs”) are the cornerstone of many successful physics classrooms. Labs give students the opportunity to investigate behaviors of objects and systems, make observations, and develop their own explanations and understandings of the physical world. Within labs, students explore patterns and systems to make conclusions that can be used to predict future outcomes. This cycle of observation, measurement, recording, analyzing, and concluding is the backbone of all science. Justifying conclusions by applying the knowledge, concepts, and principles that are discussed in lecture or classwork sessions to tangible material connects physical actions to conceptual understanding. Labs provide students with additional ways to encode information, which increases the methods by which they are able to retrieve and apply that information. An analysis of data from AP Physics examinees regarding the length of time they spent per week in the laboratory shows that increased lab time correlates with higher AP Exam scores.

## Descriptions of Labs

AP Physics courses require that 25% of instruction time is spent in hands-on laboratory work. In practice, this translates to approximately one classroom period of 45 minutes per week (for a course that meets 5 days a week for 45 minutes for the entire school year) that is devoted to lab-related activities. This average can be implemented in a variety of ways. Some teachers do labs on the same day every week, using a weekly cycle of content, practice, application, and low-stakes formative assessment. Others prefer to do one “big” or more complex lab every two weeks. Some teachers prefer to thread quick 20-minute “mini labs” throughout their daily classroom routines. Some teachers plan a few investigative labs at the start of a unit; then spend significant class time on practice, discussion, and application of concepts; and then end the unit with one or two more complex labs that incorporate content from the entire unit. And even further still, some teachers do a combination of all of the above. Any approach is acceptable as long as the 25% requirement is met.

## Defining “Labs” and “Lab Time”

Perhaps the most common questions asked by teachers who are planning their lessons are variations of:

- “What counts as a lab?”
- “What counts as time spent on labs?”
- “Do labs need to take the entire class period?”
- “Do students need to be in the ‘lab’ part of the classroom to count as lab time?”
- “Do digital or online lab simulations count for lab time?”
- “Do labs need to have formal write-ups?”

## Labs

For AP Physics, a *lab* is performed any time data is collected and/or analyzed. A follow-up question might then be: “What is the threshold for collecting data?” For AP Physics, data is collected any time a student writes down an observation or measurement. Data can be qualitative, such as “The balloons repel each other” or quantitative. Data can be recorded in many ways, such as tables, lists, or paragraphs. The analysis itself can also be qualitative or quantitative, as appropriate for the objective of the lab.

## Lab Time

For AP Physics, *lab time* is any time spent in the classroom that supports the act of doing a lab, as defined above. This includes, but is not limited to:

- Time spent discussing the goal and objectives of a lab.
- Setup of equipment and lab stations.
- “Pre-lab” questions and activities, such as identifying what to measure, developing and writing experimental procedures, sketching lab set-ups, and creating data tables to complete.
- “Performing experimental procedures.
- Collecting, plotting, and analyzing data as needed.
- “Post-lab” activities, such as interpreting, comparing, and discussing the results of the lab, connecting these results to course content, or exploring extension questions.
- Cleanup of equipment and lab stations.

Note that all of the above can be done individually, in small lab groups, or as an entire class, as deemed appropriate for any given teacher's students and class within the context of a specific lab. There are times when it is appropriate to simply let students explore on their own, and other times when more specific instructions and directions are required (either because of complexity or safety, or both). Sometimes a long summary and review session is not needed; other labs benefit from having the entire class share their data and make conclusions using this larger pool of data. All this time spent supporting the lab may be counted as *lab time*.

## Types of Labs

Most labs can be broadly categorized into three types: Investigations, Verifications, and Applications.

*Investigation labs* are activities where students are asked to induce an outcome and discover mathematical relationships or qualitative properties without having been taught the answer in the classroom. For example, a lab may pose the question: "What is the relationship between the current in a circuit element, the resistance of the element and the potential difference across the element?" Prior to this lab, teachers will have given definitions of current, resistance and potential difference, but may not have discussed Ohm's Law. Students will determine what data to record, how to obtain that data, take measurements, and then make a conclusion about the relationship between the current, resistance and potential difference. Note that if students already know Ohm's Law, an investigation lab is easily turned into a verification lab.

*Verification labs* confirm information students have already been provided. Depending on the approach of the teacher, these labs can also be beneficial and useful in the classroom. However, the power of self-discovery should be harnessed as often as possible, as it frequently helps students make their learning more permanent than when they simply confirm the answer found by others.

An *application lab* is when students are asked to apply a known physical principle or idea to a lab setup in order to find a specific answer or quantity. For example, a lab may ask students to "Determine the resistance of a resistor by measuring current in the resistor and the potential difference across the resistor." For this lab, students already know that and apply that relationship to accomplish the objective of the lab. The results in application labs are typically easy to assess as "right" or "wrong". If students experimentally determine that the resistance of the resistor is  $x$ , and the given resistance of the resistor

is  $x$ , that provides instant feedback on how well the students applied physics, laboratory skills, and the quality of their measurements. A lab group that obtains  $x$  for the same resistor has similar instant feedback.

In an application lab, an unexpected result that significantly departs from the expected result should lead to double-checking measurements and procedures, recalibration of equipment, or finding other errors in methods or data. The ability for students to use instant feedback to check the accuracy of their work can be an invaluable tool used to develop student confidence as well as refine skills and understanding. Even when students do not obtain the "right" answer the first time, a student can earn a tremendous feeling of pride and accomplishment after self-correcting. Not only does this student learn how to overcome their mistakes, but the learning may also then be associated with positive outcomes.

## Lab Skills

Labs should be selected to implement a wide variety of appropriate scientific skills. Within the context of AP Physics, all three skills within Science Practice 3: Scientific Questioning & Argumentation are appropriate to emphasize. However, in performing an experiment, students may also demonstrate any of the other skills within the AP Physics Science Practices. As such, teachers should intentionally choose which skills to emphasize, and when. For instance, some labs may require meticulous measurement and data collection, while others only need qualitative observations. Teachers are encouraged to choose labs that represent all the skills students will need to become well-rounded scientists and physicists. No single type of lab or instructional approach can provide a one-size-fits-all solution for students in the classroom. Students benefit from a variety of strategies and approaches to gain a deep, comprehensive understanding of physics concepts.

Suggested lab variations to address a variety of course skills include:

- Labs that have small details that require careful attention as well as labs that are basic and straightforward.
- Labs that require students to repeat the same measurement many times to find an average as well as labs where students must change a single quantity in order to find a relationship between two variables.
- Labs that require graphical linearization and complex mathematical derivation to determine an experimental value as well as labs that have simple conclusions based on qualitative observations.

## Lab Formats

Labs may appear in a variety of formats. For a quick 15-minute demonstration where students make observations and conclusions, students may simply write their observations on the front of an index card and their conclusions on the back side of that same card. There is no one right way to do labs in the classroom; there are many different methods that can be employed. Some suggested lab formats include:

- Printing out pages for students to complete
- Requiring students to keep detailed laboratory notebooks
- A combination approach, including both printed workbook style pages and full laboratory notebooks
- Traditional lab manual and full lab report style

Teachers are encouraged to use routine to their advantage in the lab. If structures and routines are introduced at the start of the academic year, the complexity of labs that can be performed throughout the course can be increased. At the start of the year, teachers can reinforce and emphasize the lab skills themselves (developing procedures, recording data effectively, analysis methods, etc.) in the context of more accessible content. For example, if the goal of a lab is to analyze the speed of a bowling ball as it rolls down a hallway, students can typically develop and refine a scientific method of doing so without having to use complex experimental equipment and setups or nuanced physics. Later in their studies, a lab could have students investigate the relationship between the density of a fluid and the acceleration of an object through that fluid. While this lab would require specific equipment as well as more complex data collection and analysis, students would be able to focus on these complexities because they have practiced the foundational lab skills and techniques throughout the course.

## Lab Equipment

There is no required lab equipment in AP Physics. It is possible to develop a robust series of labs that only require the most basic materials. However, there is a wide range of equipment, sensors, and tools that are available to teachers to use to augment their current compilation of materials. A list of the most commonly used lab equipment is provided below. While students do not need to have personal experience with each of these tools, they need to be made aware of this equipment, as well as the uses for each piece, so that they will be aware of what equipment they might be asked about on the AP Exam.

## Generic Lab Equipment

- Stands, 90-degree cross supports, and clamps, of varying sizes and design
- Metersticks
- Scale (digital, triple-beam balance, etc.)
- String
  - ♦ Often heavy-duty fishing line is the best “physics” string. Fishing line is inexpensive, thin, easily available, and strong. Cotton twine is often too thick or breaks too easily, and nylon string can be difficult to tie in a reliable knot.
- Tape (including masking tape, painter’s tape, duct tape, clear tape, etc.)
- Scissors
- Glue and/or glue sticks
- Assorted spheres, balls, and masses

## Physics-Specific Optional Lab Equipment

- Lightbulbs
- Wires
- Batteries
- Switches
- Resistors
- Capacitors
- Compass
- Magnets
- Ohmmeter
- Ammeter
- Voltmeter
- Capacitance meter
- Electrostatics kit (Rubber and plastic rods, fur, silk, etc.)
- Pith balls
- Coils
- Mini generators
- Magnetic field meter
- Conductive paper
- Variable power supply
- Cell Phones

Modern cell phones are packed with sensors, cameras, and apps that can be easily and appropriately implemented in the classroom. Cell phones have stopwatches, video cameras (and most with slow-

motion or even time-lapse capabilities), accelerometers that can be used to measure angles and acceleration, magnetometers, microphones, GPS, etc. Additionally, there are many free apps that students can download that take advantage of these sensors within a physics lab setting. Cell phones can be used if available but are not required. For most laboratory investigations where data collection with phones is desired, one phone per group will suffice.

## Video Analysis/Computer Software

There are multiple options for computer software and video analysis of varying cost and functionality. Electronic sensors should come with the software needed to operate those sensors. More robust software can use collected data to measure the area under a curve, or perform curve-fitting analyses or linear regressions, and more.

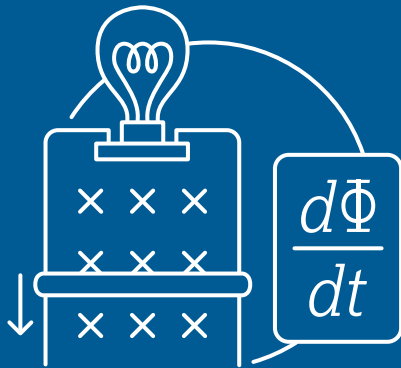


THIS PAGE IS INTENTIONALLY LEFT BLANK.

**AP PHYSICS C: ELECTRICITY AND MAGNETISM**

---

# Instructional Approaches



# Selecting and Using Course Materials

---

## Selecting and Using Course Materials

Teachers will benefit from a wide array of materials to help students become proficient with the science practices necessary to develop a conceptual understanding of the relationships, laws, and phenomena studied in AP Physics C: Electricity and Magnetism. In addition to using a college-level textbook that will provide required course content, teachers should provide students with regular opportunities to create and use data, representations, and models through supplemental material such as TIPERS (Tasks Inspired by Physics Education Research), AP Classroom, and released AP Physics C: Electricity and Magnetism questions. Rich, experimental investigation is the cornerstone of AP Physics C: Electricity and Magnetism, and diverse source material allows teachers more flexibility in designing the types of learning activities that will help develop the habits of thinking like a physicist.

## Textbooks

While nearly all college-level physics textbooks address the six units of AP Physics C: Electricity and Magnetism, it's important for teachers to identify other types of secondary sources (such as lab manuals, TIPERS, etc.) to supplement the chosen textbook, accordingly, ensuring that each of the six topic areas, as well as the science practices, receives adequate attention. AP Central provides an example textbook list to help determine whether a text is considered appropriate in meeting the AP Physics C: Electricity and Magnetism Course Audit resource requirement. Teachers can also select textbooks locally.

# Guided Inquiry in AP Physics C: Electricity and Magnetism

The more active students are in their science education, the more scientifically literate they will become. Inquiry into authentic questions generated from student experiences should be one of the central strategies when teaching AP Physics C: Electricity and Magnetism. By posing questions, planning investigations to answer those questions, and reviewing what is already known

in light of experimental evidence, students mirror how scientists analyze the physical world. Inquiry requires identifying assumptions, using critical and logical thinking, and considering alternative explanations. Having students probe for answers to scientific questions will lead to a deeper understanding of scientific concepts.

Skill	How to Scaffold Inquiry in the AP Classroom			
	MORE	← AMOUNT OF DIRECTION FROM TEACHER →		LESS
<b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.	The student works with a representation provided by the teacher.	The student selects from a set of representations provided by the teacher.	The student sharpens, completes, or augments a representation provided by the teacher.	The student creates their own representation.
<b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.	The student works with a graph provided by the teacher.	The student plots data on a graph that is labeled and scaled, but not fully completed, by the teacher.	The student applies appropriate scales and plots data on a graph where the axes have been labeled for the student by the teacher.	The student creates appropriate graphs with scales and units, including plotting data, on their own.
<b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.	The student works with sketches of graphs that are provided by the teacher.	The student is able to identify important characteristics of a sketch of a given scenario (i.e., whether the quantity should be increasing or decreasing but not necessarily whether it is concave up or down).	The student creates a sketch that represents a feature of a model, based on a sketch from a slightly different scenario, either provided by the teacher or reviewed previously.	The student creates sketches of graphs that represent features of a model or the behavior of a system, on their own.
<b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.	The student works with a derivation provided by the teacher.	The student selects from a set of given derivations.	The student is given possible starting points for a derivation by the teacher.	The student derives an expression using physics concepts and principles on their own.

## How to Scaffold Inquiry in the AP Classroom

Skill	<b>MORE</b> ← <b>AMOUNT OF DIRECTION FROM TEACHER</b> → <b>LESS</b>			
<b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.	The student works with a calculation provided by the teacher.	The student selects from a set of given calculations provided by the teacher.	The student is given possible starting points for a calculation by the teacher.	The student performs a calculation on their own using physics concepts and principles.
<b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.	The student is given the relationship or pattern between quantities from the teacher.	The student is given possible relationships or patterns to choose from to compare quantities from the teacher.	The student is directed toward possible patterns or relationships by the teacher.	The student can examine relationships and form links to explanations on their own.
<b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.	The student is given a scenario and an equation by the teacher and is instructed how to analyze the scenario using the given equation.  The student is given data by the teacher and is told how to analyze it.	The student is given possible equations or relationships between variables for a given scenario by the teacher and is asked to choose the equation or relationship that could be helpful in analyzing the scenario.  The student is given data to analyze by the teacher.	The student is directed toward first principles by the teacher that will help start a derivation to make claims about the functional dependence between variables in a given scenario.  The student is directed to collect and analyze certain data by the teacher.	The student can derive relationships and make claims about the functional dependence between variables in a given scenario on their own.  The student can determine both what constitutes evidence to support a claim and collect it on their own.
<b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.	The student works with a procedure provided by the teacher.	The student selects from a set of given procedures provided by the teacher.	The student sharpens, completes, or augments an outline of a procedure provided by the teacher.	The student determines a procedure on their own.
<b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.	The student is given procedures, relationships, or data by the teacher to make claims and predictions.	The student is given broad guidelines from the teacher to sharpen claims and predictions.	The student is coached in the development of claims and predictions using evidence summarized by the teacher.	The student devises a claim or prediction on their own after summarizing the evidence.
<b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.	The student is provided with evidence to support an explanation.	The student is given possible ways to use evidence to create explanations by the teacher.	The student is guided through the process of formulating explanations from evidence.	The student can form reasonable and logical arguments to communicate explanations based on scientific theories and models on their own.

Understanding the different types of inquiry can help teachers scaffold the types of labs and activities to better meet the needs of their students.

Below are four suggestions to make labs and activities more student-centered and inquiry-based:

- **Start small:** Take out the “data” or “results” section from traditional labs. If the procedure is thorough and simple enough, students can read and design the data and results sections on their own.
- **Tackle the procedure:** Eventually, teachers will want students to design their own experiments, but students may need some practice first. Remove the step numbers and shuffle the steps in the procedure. Have the students work in pairs to put the steps into the correct order. Next, try having them write a procedure as a pre-lab homework assignment, and then work together as a class to develop it further, making sure that the question, variables, and safety are addressed.
- **Try a goal-oriented task:** Completely remove the procedure, and prompt students with a question that asks them to achieve something they want to do. At this point, it’s best to choose a lab that students already understand conceptually and that uses simple, familiar equipment.
- **Let students do the thinking:** Create opportunities for students to choose what they will investigate. Facilitate their thought process without telling them what to do. A pre-lab brainstorming session in small groups is helpful when having students develop a question to investigate. It is important to provide students with some guidelines at this step. For example, students need to think about a question, a hypothesis, and materials before beginning an open-ended lab. Seeing and approving this in lab groups helps boost students’ confidence.

# Instructional Strategies

---

The AP Physics C: Electricity and Magnetism course framework outlines the concepts and skills students must master in order to be successful on the AP Exam. To address those concepts and skills effectively, teachers should incorporate a variety of instructional approaches and best practices into their daily lessons and activities. Teachers can help students develop mastery of the skills by engaging them in learning activities that allow them to apply their understanding of course concepts. Teachers may consider the following strategies as they plan instruction. Please note they are listed alphabetically and not by order of importance or instruction.

Strategy	Definition	Example
<b><i>Ask the Expert</i></b>	Students are assigned as “experts” on concepts they understand well; groups rotate through the expert stations to learn about concepts they need to work on, providing students with opportunities to share knowledge and learn from one another.	Assign student “experts” on Gauss’s Law questions. Have students rotate through stations in groups working with the station expert to justify a set of claims pertaining to each question with corresponding physical laws. “Experts” can be swapped at any point during the rotation so that all students have the opportunity to lead work and engage with multiple problems.
<b><i>Changing Representations</i></b>	Students translate from one representation (e.g., an electric field diagram) to another (e.g., an equipotential curve or surface diagram). This may involve creating pictures, tables, graphs, lists, equations, models and/or verbal expressions to interpret text or data.	For a given situation involving analyzing circuits, have students create a circuit diagram, a set of Kirchhoff’s rules equations, and a graph of the potential around the circuit as a function of the position in the circuit.
<b><i>Concept-Oriented Demonstration</i></b>	Students create a description, prediction, and/or explanation for a demonstration done by the teacher.	While demonstrating that bulbs of different wattage have different brightnesses depending on how they are connected in the circuit, have students explain the outcome of the experiment in terms of physical laws and theories.
<b><i>Conflicting Contentions</i></b>	Students are presented with two or three statements that disagree in some way. They then have to decide which contention they agree with and explain why.	A neutral conducting sphere encloses a charged nonconducting sphere. Two students make conflicting contention about the charge on the inner surface and outer surface of the conducting sphere. Have students identify elements of the contentions that are correct or incorrect, and justify their reasoning.

*continued on next page*



Strategy	Definition	Example
<b>Construct an Argument</b>	Students use mathematical reasoning to present assumptions about mathematical situations, support conjectures with mathematically relevant and accurate data, and provide a logical progression of ideals leading to a reasonable conclusion. This strategy can be used with scenarios presented verbally that do not lend themselves to immediate application of a formula or mathematical process.	Provide students with an electric field map of the area around two unknown charges and ask them to construct an argument either in defense of or against the claim that the two charges have the same sign.
<b>Create a Plan</b>	Students analyze the tasks in a problem and create a process for completing the tasks. They find the information needed, interpret data, choose how to solve the problem, communicate the results and verify accuracy.	Have groups of three to four students analyze the tasks necessary to design an experiment to determine whether a lightbulb can be considered an ohmic resistor by scaffolding the process. Have students identify the steps needed to determine the relationship, including collecting and analyzing data, as well as what to do with the collected data.
<b>Debriefing</b>	Students discuss the understanding of a concept to lead to a consensus on its meaning while clarifying misconceptions and deepening understanding of context.	Have students discuss their understanding of commonly misunderstood concepts such as electric potential. Have students include examples that illustrate their examples to highlight their ability to apply these concepts to physical scenarios.
<b>Desktop Experiments Tasks</b>	Students perform a demonstration at their desks (either in class or at home) using a predict - and explain format but add the step of actually doing the experiment. This “doing it” step consists of using apparatus provided to answer a given question and is followed by a reformulating step, where students reconsider their previous explanations while considering the results of the experiment.	<p>Have students fold the end of a piece of clear tape and then stick it to their desk. Repeat with a second piece of tape, but put the second piece of tape on top of the first piece of tape. Then pull both pieces of tape off the desk, and then pull the pieces of tape together.</p> <p>Students can then investigate the net charge of the pieces of tape and make conclusions based on their observations. Do the pieces of tape have the same or opposite charge? Is the top piece always the same charge? What happens if the tape touches each other? What happens as the tape is brought closer or farther away from each other?</p>
<b>Discussion Groups</b>	Students work in pairs or groups to discuss related content, create problem solutions, and explain and justify a solution. Participating in these activities aids in understanding through the sharing of ideas, interpretation of concepts, and analysis of problem scenarios.	Have students watch a video outside of class about capacitors. Ask students to take their own notes. When they return to class, have students discuss what they saw and learned from the video. They can then present a summary to the class. For a class activity, each group can be assigned to watch a video about a different topic.

*continued on next page*

Strategy	Definition	Example
<b><i>Friends Without Pens</i></b>	Students solve problems by engaging in two “rounds” of timed work—in groups and then independently. In the first round, called “friends without pens,” students are grouped together to discuss the problem but are not permitted to write anything. In the second round, called “pens without friends,” students return to their desk where they complete and finalize their responses to the problem individually, using any information they remember from their group discussion and their own knowledge of course concepts.	AP Classroom has many free-response questions and challenging multiple-choice questions for each topic, from Electrostatics to Induction. Alternatively, past exam questions provide excellent opportunities to practice this instructional strategy.
<b><i>Four-Square Problem Solving</i></b>	Students are given a scenario, perhaps one that came from a traditional, “plug-and-chug” calculation problem. They divide a sheet of paper into four quadrants. In each quadrant, students put some representation of what is going on in the problem—for example, motion maps or graphs, free-body diagrams, energy bar graphs, momentum bar graphs, mathematical models (i.e., equations with symbols), well-labeled diagrams, or written justification (i.e., two to three strong, clear sentences).	Have students do a four-square with a current-carrying wire. Students can draw a representation of the B-field generated by the wire, can sketch a graph of the B-field as a function of radial distance from the wire, can derive an expression for the magnitude of the B-field as a function of radial distance, and write a few sentence that describe the magnetic field, or what would happen if the direction of current was reversed, or perhaps if the magnitude of the current was doubled.
<b><i>Graph and Switch</i></b>	Each student in a pair generates a graph (or sketch of a graph), on a graphing calculator or on paper, to model a certain function. Then, the students switch graphing calculators or papers to review each other’s solutions.	Have students draw graphs for the current in portions of an LC or RC circuit as a function of time. Students can then compare and contrast their answers with their peers. Similarly, students could derive an expression charge on the positive plate of the capacitor as a function of time and compare their final answers.
<b><i>Marking the Text</i></b>	Students highlight, underline, and/or annotate a text to identify and focus on key information that helps them understand the concepts and interpretations of tasks required to solve the problem.	Have students read through an AP-level question on experimental design from AP Classroom - or have them look at a write up of another student’s experimental design - and underline the pronouns (especially “it”) equipment, and key information (e.g., the capacitor is initially uncharged) to identify important details needed to answer the question or improve a given response. Leave time for students to ask clarifying questions about words or phrases they find unclear before asking them to provide a solution.

*continued on next page*

Strategy	Definition	Example
<b>Meaningful, Meaningless Calculations</b>	Students decide whether a calculation is meaningful (i.e., it gives a value that tells us something legitimate about the physical situation) or is meaningless (i.e., the expression is a totally inappropriate use of a relation). For example, a meaningless calculation might involve substituting a wrong numerical value into an expression.	<p>Give students a situation, such as an electron passing through a magnetic field. Show a derivation that results in a radius of an incredibly small or incredibly large number and ask if that answer makes sense. Perhaps the radius of the circle requires the electron to travel faster than the speed of light, which is impossible. Or perhaps the required magnetic field is magnitudes greater than what is realistically possible to achieve.</p> <p>Alternatively, dimensional analysis of units can be used to determine if a given mathematical calculation will produce the desired answer.</p>
<b>Note Taking</b>	Students create a record of information while reading a text, listening to a speaker, or interacting with a problem.	Have students write down descriptions of the steps needed to solve a problem in words so that a record of the processes can be referred to at a later point in time.
<b>Predict and Explain</b>	Students predict what will happen in a situation - one they are familiar with or have sufficient background information about - and explain why they think that outcome will occur.	When a pair of capacitors is set into a circuit, what would happen to the time to charge and/or discharge the circuit if a third capacitor is set in series with the original two capacitors?
<b>Qualitative Reasoning</b>	Students are presented with an initial and a final version of the same physical situation and asked to apply a principle to qualitatively reason how some quantity, or aspect, will change.	Ask students what would happen to the electric field surrounding a uniformly charged conducting sphere if the sphere were to have twice the charge, half the radius, or has become a hollow shell. To continue their thinking, you may ask the students about the electric field inside the sphere versus inside the shell.
<b>Quickwrite</b>	Students write for a short, specific amount of time about a designated topic.	To help synthesize concepts after having learned how to calculate the derivative, students list as many real-world situations as possible in which knowing the instantaneous rate of change of a function is advantageous.
<b>Ranking Tasks</b>	Students are presented with a series of variations of a situation, based on a specific scenario. The variations differ in the values (numeric or symbolic) for the variables involved, but also frequently include variables that are not important to the task. Students rank the variations of a specified physical quantity and must also explain the reasoning for their ranking choices, as well as rate their confidence in their ranking.	Given six different circuits containing identical resistors, students are asked to rank the circuits by greatest total resistance.

*continued on next page*

Strategy	Definition	Example
<b><i>Sharing and Responding</i></b>	Students communicate with each other in pairs or in groups, taking turns proposing a solution to a problem and responding to the solutions of others.	Have students design an experimental procedure to determine the magnitude of a magnetic field by using a loop of wire. Students can work on their own or in small groups. Then the groups share their procedures with other groups, and determine if the group's procedures would produce a valid measurement. Groups can justify their analyses either to their groups or in a larger class discussion.
<b><i>Simplify the Problem</i></b>	Students use "friendlier" numbers or functions to help solve a problem.	Have students use the definition of an electric field surrounding a point charge to model the electric field around a uniformly charged conducting sphere.
<b><i>"What if Anything Is Wrong?"</i></b>	Students analyze a statement or diagrammed situation to determine if it is correct. If everything is correct, students explain why the situation/ statement works as described. If something is incorrect, students must identify the error and explain how to correct it.	Students can be provided with various representations that depict an electron moving into a magnetic field directed into the page, then into a magnetic field directed out of the page, and then out of the magnetic field. Students can identify what, if anything, is wrong with the provided free-body diagrams for each region, or velocity vs. position graphs, or drawn trajectories of the electron.
<b><i>Write and Switch</i></b>	Students make observations, collect data, or make a claim about a situation and then switch papers with a partner. Each student in the pair gives feedback on the other's work and then returns the paper.	Have students use a battery, wire, and small resistor to create a circuit. Students then use a compass around the circuit, and make and write observations. Students can experiment with adding more batteries (in series) or a larger/smaller resistor and make observations of the compass in each. Students then switch papers with a different group and compare their results and provide feedback to the other groups.
<b><i>Working Backward</i></b>	Students work with the reverse order of the steps for solving a problem. For example, the given information could be an equation with specific values for all, or all but one, of the variables. Students then construct a physical situation for which the given equation would apply.	Students are given an electric potential as a function of position around a circuit. Students have to create a circuit schematic, including values of the potential difference across the battery and values for the resistance of the resistors, that matches the provided graph. To increase the difficulty, two graphs could be provided that depict two loops within the same circuit.

# Developing the Science Practices

---

Throughout the course, students will develop skills that are fundamental to the discipline of physics. Students will benefit from multiple opportunities to develop these practices in a scaffolded manner. The tables that follow look at each of the science practices and their associated skills and provide examples of questions with sample activities for incorporating instruction on that skill into the course.

## Science Practice 1: Creating Representations

### **Create representations that depict physical phenomena.**

When physicists describe and explain complex phenomena, they try to simplify real objects, systems, and processes to make the analysis manageable. These simplifications or models are used to predict how new phenomena will occur. A simple model may treat a system as an object, neglecting the system's internal structure and behavior. More complex models are models of a system of objects, such as a firework display or planets orbiting the sun. A process can be simplified, too. Models can be both conceptual and mathematical. Kirchhoff's rules are an example of a mathematical model, while the model of positive charge carriers moving through a wire as a steady flow of particles is a conceptual model.

To make a good model, students need to identify a set of the most important characteristics of a phenomenon or system that may simplify analysis. They then need to create a representation of those characteristics. Examples of representations used to model introductory physics concepts are pictures, free-body diagrams, force diagrams, electric field diagrams, graphs, energy bar charts, vector maps, and magnetic field maps. Representations help in analyzing phenomena and making predictions and communicating ideas. AP Physics C: Electricity and Magnetism requires students to use, analyze, and/or re-express models and representations of natural or human-made systems. Students often think that to make a graph, they need to connect the data points, or that the best-fit function is always linear. Thus, it is important that they know how to construct a best-fit curve, even for data that do not fit a linear relationship.

The following table provides examples of questions and sample activities for strengthening the skill of creating representations:

## Science Practice 1: Creating Representations

Skills	Questions to Ask Students	Sample Activities
<p><b>1.A</b> <i>Create diagrams, tables, charts or schematics to represent physical situations.</i></p>	<ul style="list-style-type: none"><li>What kind of model or representation would be appropriate for this physical scenario?</li><li>What physical characteristics can be modeled or represented for this physical scenario.</li><li>What features of the representation provide information relevant to the question or problem?</li></ul>	<p>Have students divide their paper into four quarters. In each quarter of the paper, have students create a representation of a provided physical situation. For example, have students create four different representations of a charged object that passes through both an electric and a magnetic field. Representations can include graphs, equations, narratives, bar charts, motion maps, free-body diagrams, or sketches of physical situations. Have students describe the consistency between the representations</p>
<p><b>1.B</b> <i>Create quantitative graphs with appropriate scales and units, including plotting data.</i></p>	<ul style="list-style-type: none"><li>What data should be plotted?</li><li>What scale and axis labels should be used? What does an appropriately scaled graph look like?</li><li>What does a graph need to contain to be considered "correctly labeled"?</li><li>How should the data be graphed so that the best-fit curve shows a relationship?</li><li>What do the data on the graph show?</li><li>Is there a pattern present in the data? How do you know? What does the pattern show about the relationship between quantities?</li><li>What data would need to be graphed to create a linear relationship?</li><li>What is the physical meaning of the slope and/or area underneath the linearized graph?</li><li>What is the physical meaning of the y and/or x intercepts of the linearized graph?</li></ul>	<p>Have students connect a long coil of wire (such as a long spring toy from a toy store) with a constant number of turns to a power supply or lantern battery. Measure the magnetic field coil of wire as a function of the length of the coil. Ask students what properties can be determined from the known values and the slope of the resulting graph.</p>
<p><b>1.C</b> <i>Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.</i></p>	<ul style="list-style-type: none"><li>What are the main functional relationships needed to represent the phenomena?</li><li>What is the relationship between the two physical quantities?</li></ul>	<p>Have students draw the electric field as a function of position for a configuration of charges in a straight line. Have students draw the corresponding electric potential as a function of position. Then use appropriate connections between the two to describe why the two graphs are consistent with each other.</p>

## Science Practice 2: Mathematical Routines

### Conduct analyses to derive, calculate, estimate, or predict physical phenomena.

Physicists commonly use mathematical representations to describe and explain phenomena, as well as to solve problems. When students work with these representations, they should understand the connections between the mathematical descriptions, the physical phenomena, and the concepts represented in them. When using equations or mathematical representations, students need to be able to justify why using a particular equation to analyze a situation is useful and be aware of the conditions under which the equations can be used. When solving a problem, students need to be able to describe the given situation in multiple ways, including through pictorial

representations and force diagrams, and then choose an appropriate mathematical representation-, instead of first choosing a formula whose variables seem to match the givens in the problem.

Students should also be able to work with the algebraic form of an equation before substituting values, as well as be able to solve the equation and interpret the answer in terms of units and limiting case analysis. Students should be able to translate between functional relationships in equations (e.g., proportionalities and inverse proportionalities) and cause-and-effect relationships in the physical world, while also being able to evaluate a numerical result for appropriateness for the given context.

The following table provides examples of questions and instructional strategies for implementing mathematical routines into the course:

### Science Practice 2: Mathematical Routines

Skills	Questions to Ask Students	Sample Activities
<p><b>2.A</b></p> <p><i>Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.</i></p>	<ul style="list-style-type: none"> <li>What laws, definitions, or mathematical relationships relate to the given problem?</li> <li>What are the rules, assumptions, or limitations surrounding the use of the chosen law, definition, or relationship?</li> <li>Did the derivation begin with an equation or a fundamental physics relationship, law, or definition? If so, which one?</li> <li>Are the steps clearly written out and annotated? Are any steps skipped? If so, which ones?</li> </ul>	<p>Have students derive the terminal velocity of a conducting rod that falls on connected vertical rails within a magnetic field.</p>
<p><b>2.B</b></p> <p><i>Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.</i></p>	<ul style="list-style-type: none"> <li>Did the calculation begin with an equation or a fundamental physics relationship, law, or definition? If so, which one?</li> <li>What known quantities can be used to calculate the unknown quantity?</li> <li>What steps should you follow to use the known quantities to calculate the unknown quantity?</li> <li>How should you label the calculated quantity? What units should be used?</li> </ul>	<p>Have students calculate the resistance of a bar falling through a magnetic field, given the magnitude of the magnetic field and the terminal velocity of the bar.</p>

*continued on next page*

## Science Practice 2: *Mathematical Routines*

Skills	Questions to Ask Students	Sample Activities
<b>2.C</b> <i>Compare physical quantities between two or more scenarios or at different times and/or locations within a single scenario.</i>	<ul style="list-style-type: none"><li>What relationship(s) link the needed and given quantities?</li><li>Can the relationship be rewritten so that the variable in question is alone on one side of the equation?</li><li>What quantities in the relationship are constants versus variables that can change?</li></ul>	Have students current in two bars of different masses that fall vertically within a magnetic field at terminal velocity.
<b>2.D</b> <i>Predict new values or factors of change of physical quantities using functional dependence between variables.</i>	<ul style="list-style-type: none"><li>What relationship(s) link the needed and given quantities?</li><li>What are the rules, assumptions, or limitations surrounding the use of the chosen law, definition, or relationship?</li><li>How are the quantities in the relationship related (e.g., directly and inversely)?</li><li>What words would you use to describe the functional dependence of the variables on each other?</li></ul>	Have students determine the factor by which the potential difference across a rod falling vertically in a magnetic field would change if the length of the bar was doubled.



## Science Practice 3: Scientific Questioning and Argumentation

### Describe experimental procedures, analyze data, and support claims.

Physicists examine data and evidence to develop claims about physical phenomena. As they articulate their claims, physicists use reasoning processes that rely on their awareness of different types of relationships, connections, and patterns within the data and evidence. They then formulate a claim and develop an argument that explains how the claim is supported by the available evidence. As a result, students should learn how to create persuasive and meaningful arguments by using claims they develop and evidence they've identified to support those claims.

Scientific questions can range in scope as well as in specificity, from determining influencing factors to determining mechanisms. The question posed will determine the type of data to be collected and will influence the plan for collecting data. Designing and improving experimental designs and/or data-collection strategies is a learned skill. Class discussions can reveal

issues of measurement uncertainty and assumptions in data collection. Being able to devise testable explanations goes hand-in-hand with dealing with new phenomena. It is important that students understand that scientific instruments do not produce exact measurements and learn what steps they can take to decrease uncertainty. One step may be designing a second experiment to determine the same quantity and then checking for consistency across the two measurements. Finally, students should be able to revise their initial explanation or reasoning based on the new data. The analysis, interpretation, and application of quantitative information are vital skills for students in AP Physics C: Electricity and Magnetism. Analysis skills can be taught using any type of data, but students will be more invested in their data analysis if it is data they have collected through their own investigations. Students should be encouraged to analyze their data, draw conclusions, and apply their knowledge to content across the course.

The following table provides examples of strategies for implementing opportunities to practice argumentation throughout the course:

### Science Practice 3: *Scientific Questioning and Argumentation*

Skills	Questions to Ask Students	Sample Activities
<p><b>3.A</b> <i>Create experimental procedures that are appropriate for a given scientific question.</i></p>	<ul style="list-style-type: none"> <li>▪ What information will be needed to answer the scientific question? What data should be collected?</li> <li>▪ What equipment is needed to collect the necessary data?</li> <li>▪ How will each piece of equipment be used to collect the necessary data?</li> <li>▪ What possible errors need to be addressed before data collection?</li> <li>▪ What steps can be taken to decrease the uncertainty in the measurements and data?</li> <li>▪ What changes can be made to observations and measurements to refine the data?</li> <li>▪ How will the data be analyzed to answer the scientific question?</li> <li>▪ How can a second experiment be designed to answer the same scientific question and check for consistency?</li> </ul>	<p>Have students design an experiment designed to determine how capacitance changes as a function of the distance between the plates, given a set of household materials, such as a 9 volt battery, wires, aluminum foil, and a book.</p>

*continued on next page*

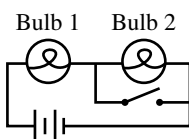
### Science Practice 3: Scientific Questioning and Argumentation

Skills	Questions to Ask Students	Sample Activities
<b>3.B</b> <i>Apply an appropriate law, definition, theoretical relationship, or model to make a claim.</i>	<ul style="list-style-type: none"><li>What law, definition, relationship, or model can be used to make a claim about the scenario?</li><li>What is your purpose (e.g., to define, show causality, compare, or explain a process) for making a claim?</li></ul>	Have students determine if a shell surrounding a charged sphere is conducting or nonconducting, based on a provided electric field as a function of radius graph.
<b>3.C</b> <i>Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.</i>	<ul style="list-style-type: none"><li>What reasoning (e.g., physical laws, theories) supports your claim? How does the reasoning support your claim?</li><li>How does the evidence support your claim?</li></ul>	Give students two graphs of current as a function of time for a point within two RC circuits. One graph starts at a nonzero positive value and decays to zero. The second graph starts at a larger nonzero positive value and decays to zero in less time. Tell students the resistor in both circuits is identical. Ask students what physical features of the capacitor could be different between each circuit, and justify that claim with evidence from the graphs.

## Practicing with Science Practices and Skills:

### CASE STUDY—CIRCUITS

The following multiple choice questions all use the same stimulus and basic scenario. However, each multiple-choice question is written to assess a different course skill. This case study helps illustrate how the same content can be assessed in the context of different skills. The more opportunities that students have to practice content with different science practices, the better prepared they will be for the AP Physics exam. The content below is appropriate for AP Physics C: Electricity and Magnetism (Science Practice 1 is FRQ only, and so is not represented below.)



A circuit contains two identical lightbulbs and a switch connected to a battery, as shown. The switch is initially open.

### Skill 2.A—Derivations

Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

#### QUESTION 2.A

The bulbs each have a resistance  $R$ , and the emf supplied by the battery is  $\mathcal{E}$ . The switch is open. Which of the following expressions correctly represents the power delivered to Bulb 1?

(A)  $P = \frac{2\mathcal{E}^2}{R}$

(B)  $P = \frac{\mathcal{E}^2}{R}$

(C)  $P = \frac{\mathcal{E}^2}{2R}$

(D)  $P = \frac{\mathcal{E}^2}{4R}$

### Skill 2.B—Calculations

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

#### QUESTION 2.B

The bulbs each have a resistance of  $2.0 \, \Omega$ , and the emf supplied by the battery is  $9.0 \, \text{V}$ . The switch is open. Which of the following is most nearly the power delivered to Bulb 1?

(A) 10 W

(B) 20 W

(C) 40 W

(D) 80 W

### Skill 2.C—Comparisons

Compare physical quantities between two or more scenarios or at different times and/or locations within a single scenario.

#### QUESTION 2.C

When the switch is open, the bulbs have equal brightness. When the switch is closed, which of the following occurs to the bulbs?

	Bulb 1	Bulb 2
A	Gets brighter	Gets brighter
B	Gets brighter	Goes out
C	Gets dimmer	Goes out
D	Gets dimmer	Gets brighter

(A) A

(B) B

(C) C

(D) D

## Skill 2.D—Functional Dependence

Predict new values or factors of change of physical quantities using functional dependence between variables.

### QUESTION 2.D

When the switch is open, the current in Bulb 1 is  $I_o$ . When the switch is closed, the current in Bulb 1 is  $I_c$ . Which of the following correctly relates  $I_o$  to  $I_c$ ?

- (A)  $I_o = 2I_c$
- (B)  $I_o = I_c$
- (C)  $I_o = \frac{1}{2}I_c$
- (D)  $I_o = \frac{1}{4}I_c$

## Skill 3.B—Make a Claim

Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

### QUESTION 3.B

Which of the following claims about the arrangement of the circuit is correct?

- (A) The two bulbs are in series with each other.
- (B) Bulb 2 is in parallel with the battery.
- (C) Bulb 1 is in parallel with the battery.
- (D) Bulb 2 is in series with the switch.

## Skill 3.C—Justify a Claim

Support a claim using evidence from experimental data, physical representations, or physical principles or laws.

### QUESTION 3.C

The switch is closed, and Bulb 2 is replaced with an uncharged capacitor of capacitance  $C$ . The switch is then opened. Which of the following describes the current in the battery after the switch is closed, and provides a correct justification?

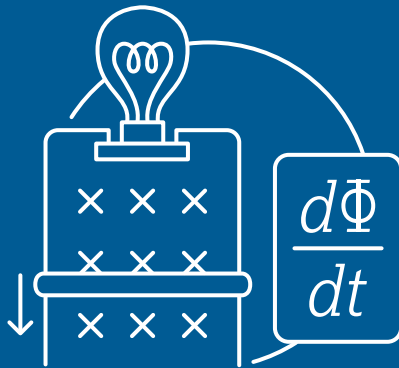
- (A) The current in the battery is constant because batteries always provide constant current.
- (B) The current in the battery is constant because the capacitor and resistor are in series.
- (C) The current in the battery decreases because energy is dissipated by the resistor.
- (D) The current in the battery decreases because the potential difference across the resistor decreases as the capacitor is charged.

THIS PAGE IS INTENTIONALLY LEFT BLANK.

**AP PHYSICS C: ELECTRICITY AND MAGNETISM**

---

# Exam Information



# Exam Overview

---

*The AP Physics C: Electricity and Magnetism Exam assesses student application of the science practices and understanding of the learning objectives outlined in the course framework. The exam is 3 hours long and includes 40 multiple-choice questions and 4 free-response questions. A four-function, scientific, or graphing calculator is allowed on both sections of the exam. The details of the exam, including exam weighting and timing, can be found below:*

Section	Type of Questions	Number of Questions	Weighting	Timing
<b>I</b>	Multiple-choice questions	<b>40</b>	<b>50%</b>	<b>80 minutes</b>
<b>II</b>	Free-response questions	<b>4</b>	<b>50%</b>	<b>100 minutes</b>

Question 1: Mathematical Routines
Question 2: Translation Between Representations
Question 3: Experimental Design and Analysis
Question 4: Qualitative/Quantitative Translation



The exam also assesses each of the six units of instruction with the following exam weightings on the multiple-choice section of the AP exam:

### Exam Weighting for the Multiple-Choice Section of the AP Exam

Units of Instruction	Exam Weighting
<b>Unit 8:</b> Electric Charges, Fields, and Gauss's Law	<b>15–25%</b>
<b>Unit 9:</b> Electric Potential	<b>10–20%</b>
<b>Unit 10:</b> Conductors and Capacitors	<b>10–15%</b>
<b>Unit 11:</b> Electric Circuits	<b>15–25%</b>
<b>Unit 12:</b> Magnetic Fields and Electromagnetism	<b>10–20%</b>
<b>Unit 13:</b> Electromagnetic Induction	<b>10–20%</b>

# How Student Learning Is Assessed on the AP Exam

## Exam Weighting by Science Practice

Science Practices 2 and 3 are assessed in the multiple-choice section with the following weighting (Science Practice 1 will not be assessed in the multiple-choice section). Science Practices 1, 2 and 3 are all assessed in the free-response section with the following weighting.

Please note: Required course content (Learning Objectives and Essential Knowledge) can be assessed with any skill.

Science Practice	Approximate MCQ Exam Weighting	Approximate FR Exam Weighting
<b>1.A</b> Create diagrams, tables, charts, or schematics to represent physical situations.		
<b>1.B</b> Create quantitative graphs with appropriate scales and units, including plotting data.	<b>N/A</b>	<b>20–35%</b>
<b>1.C</b> Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.		
<b>2.A</b> Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.	<b>25–30%</b>	
<b>2.B</b> Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.	<b>20–25%</b>	<b>40–45%</b>
<b>2.C</b> Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.	<b>10–15%</b>	
<b>2.D</b> Predict new values or factors of change of physical quantities using functional dependence between variables.	<b>10–15%</b>	

Science Practice	Approximate MCQ Exam Weighting	Approximate FR Exam Weighting
<b>3.A</b> Create experimental procedures that are appropriate for a given scientific question.	N/A	
<b>3.B</b> Apply an appropriate law, definition, theoretical relationship, or model to make a claim.	15–25%	30–35%
<b>3.C</b> Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.	5–10%	

## Free-Response Questions

The free-response section of the AP Physics C: Electricity and Magnetism exam consists of four question types listed below in the order they will appear on the exam.

## Mathematical Routines (MR)

Science Practices: **1.A** **1.C** **2.A** **2.B** **3.B** **3.C**

10 Points; suggested time: 20–25 minutes

The Mathematical Routines question (MR), assesses students' ability to use mathematics to analyze a scenario and make predictions about that scenario. Students will be expected to create and use representations that describe the scenario, either to help guide the mathematical analysis (such as drawing a free-body diagram) or that are applicable to the scenario (such as sketching a graph of velocity as a function of time).

## Translation Between Representations (TBR)

Science Practices: **1.A** **1.C** **2.A** **2.D** **3.B** **3.C**

12 Points; suggested time: 25–30 minutes

The Translation Between Representations question (TBR) assesses students' ability to connect different representations of a scenario. Students will be expected to create a visual representation that describes a given scenario. Students will derive equations that are mathematically relevant to the scenario. Students will draw graphs that relate quantities within the scenario. Finally, students will be asked to do any one of the following:

- Justify why their answers to any two of the previous parts do/do not agree with each other.
- Use their representations, mathematical analysis, or graphs to make a prediction about another situation and justify their prediction using that reasoning or analysis.
- Use their representations, mathematical analysis, or graph to make a prediction about how those representations would change if properties of the scenario were altered and justify that claim using consistent reasoning or analysis.

## Experimental Design and Analysis (LAB)

Science Practices: **1.B** **2.B** **2.D** **3.A**

10 Points; suggested time: 25–30 minutes

The Experimental Design and Analysis question (LAB) assesses students' ability to create scientific procedures that can be used with appropriate data analysis techniques to determine the answer to given questions. The LAB question can roughly be divided into two sections: Design and Analysis. In the Design portion of the LAB question, students will be asked to develop a method by which a question about a given physical scenario could be answered. Then experimental procedure is expected to be scientifically sound: vary a single parameter, and measure how that change affects a single characteristic. Methods must be able to be performed in a typical high school laboratory. Measurements must be made with realistically obtainable equipment or sensors. Students will be expected to describe a method by which the collected data could be analyzed in order to answer the posed question, by either graphical or comparative analysis.

Students will then be given experimental data collected in order to answer a similar, but not identical, equation to what was asked in the Design portion of the question. Students will be asked to use the data provided to create and plot a graph that can be analyzed to determine the answer to the given question. For instance, the slope or intercepts of the line may be used to determine a physical quantity or perhaps the nature of the slope would answer the posed question.

## Qualitative/Quantitative Translation (QQT)

Science Practices: **2.A** **2.D** **3.A** **3.C**

8 points; suggested time: 15–20 minutes

The Qualitative/Quantitative Translation question (QQT) assesses a students' ability to connect the nature of the scenario, the physical laws that govern the scenario, and the mathematical representations of that scenario to each other. Students will be asked to make and justify a claim about a given scenario, as well as derive an equation related to that scenario. Finally, students will be asked to do any one of the following:

- Justify why their answers to any of the previous parts do/do not agree with each other.
- Use their representations or mathematical analysis to make a prediction about another situation and justify their prediction using that reasoning or analysis.
- Use their representations and mathematical analysis to make a prediction about how those representations would change if properties of the scenario were altered and justify that claim using consistent reasoning or analysis.

While students may not be directly assessed on their ability to create diagrams or other representations of the system to answer the QQT, those skills may still help students to answer the QQT. For instance, some students may find that drawing a free-body diagram is useful when determining the acceleration of the system. However, the students will earn points for the explanation and conclusions that diagram indicates (or perhaps the derivation that results from the diagram), rather than for creating the diagram itself.

# Task Verbs Used in Free-Response Questions

---

The following task verbs are commonly used in the free-response questions.

**Calculate:** Perform mathematical steps to arrive at a final answer, including algebraic expressions, properly substituted numbers, and correct labeling of units and significant figures.

**Compare:** Provide a description or explanation of similarities and/or differences.

**Derive:** Starting with a fundamental law or relationship, perform a series of mathematical steps to arrive at a final answer.

**Describe:** Provide the relevant characteristics of a specified topic.

**Determine:** Make a decision or arrive at a conclusion after reasoning, observation, or applying mathematical routines (calculations).

**Draw:** Create a diagram or schematic that illustrates relationships, depicts physical objects, or demonstrates consistency between different types of representation. Labels may or may not be required.

**Estimate:** Roughly calculate numerical quantities, values (greater than, equal to, less than), or signs (negative, positive) of quantities based on experimental evidence or provided data. When making estimations, showing steps in calculations are not required.

**Indicate:** Provide information about a specified topic, without elaboration or explanation.

**Justify:** Provide qualitative reasoning beyond mathematical derivations or expressions to support, qualify, or defend a claim.

**Label:** Provide labels indicating unit, scale, and/or components in a diagram, graph, model, or representation.

**Plot:** Draw data points in a graph using a given scale or indicating the scale and units, demonstrating consistency between different types of representations.

**Rank:** Arrange quantities in relation to each other, typically by size or magnitude.

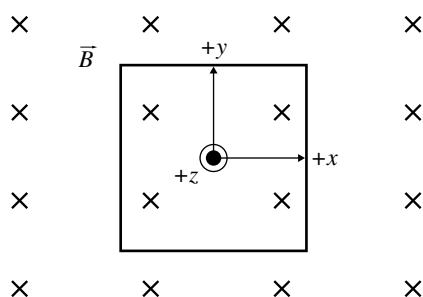
**Sketch:** Create a diagram, graph, representation, or model that illustrates or explains relationships or phenomena, demonstrating consistency between different types of representations. Labels may or may not be required.

**Verify:** Confirm that the conditions of a scientific definition, law, theorem, or test are met to explain why it applies in a given situation. Also, use empirical data, observations, tests, or experiments to prove, confirm, and/or justify a hypothesis.

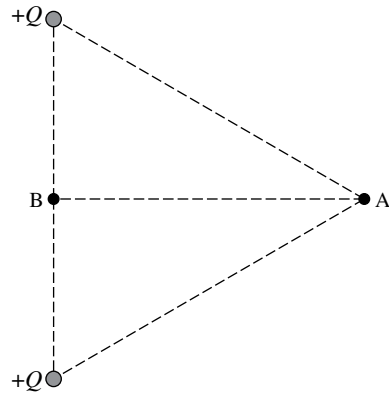
# Sample Exam Questions

The sample exam questions that follow illustrate the relationship between the course framework and the AP Physics C: Electricity and Magnetism Exam and serve as examples of the types of questions that appear on the exam. These sample questions do not represent the full range and distribution of items on an official AP Physics C: Electricity and Magnetism Exam. After the sample questions is a table which shows which skill, learning objective, and essential knowledge statement each question relates to. This table also provides the answers to the multiple-choice questions.

## Section I: Multiple-Choice Questions



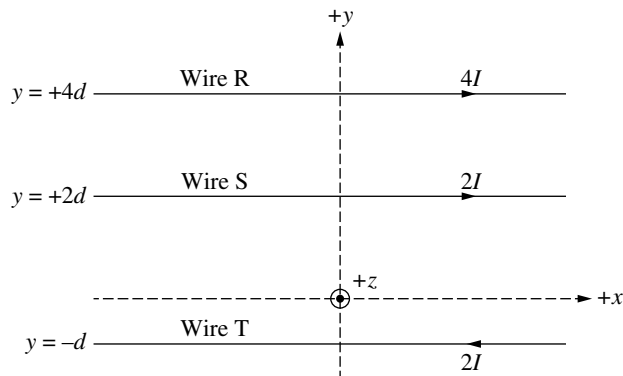
1. A rectangular loop of wire lies in the  $xy$ -plane in a region with a uniform magnetic field  $\vec{B}$  directed in the  $-z$ -direction, as shown. In which of the following cases will an emf be induced in the loop?
  - (A) When the loop is moving without rotating in the  $+y$ -direction
  - (B) When the loop is moving without rotating in the  $+z$ -direction
  - (C) When the loop is rotating about an axis along the  $z$ -axis
  - (D) When the loop is rotating about an axis along the  $y$ -axis
2. A cylindrical wire segment with length  $0.25\text{ m}$  and diameter  $3.0 \times 10^{-3}\text{ m}$  dissipates energy at a rate of  $6.0 \times 10^{-4}\text{ W}$  when a current of  $0.50\text{ A}$  is in the wire segment. Which of the following is approximately equal to the resistivity of the wire?
  - (A)  $6.8 \times 10^{-8}\ \Omega \cdot \text{m}$
  - (B)  $9.0 \times 10^{-5}\ \Omega \cdot \text{m}$
  - (C)  $7.5 \times 10^{-3}\ \Omega \cdot \text{m}$
  - (D)  $840\ \Omega \cdot \text{m}$



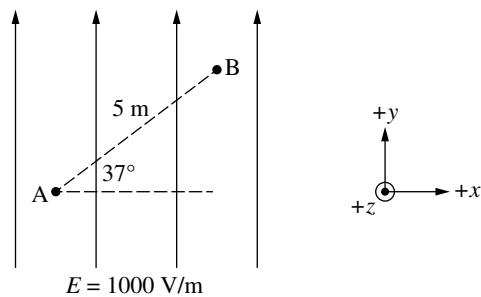
3. Two small spheres, each with positive charge  $+Q$ , are fixed in place at two corners of an equilateral triangle, as shown in the figure. Point A is at the other corner, and Point B is midway between the spheres. A test charge with positive charge  $+q$  is moved from Point A to Point B at a constant velocity by an external force. The work done by the external force on the moving particle is  $W_{\text{ext}}$ , and the work done by the electrostatic forces from the spheres on the moving particle is  $W_{\text{elec}}$ . Which of the following correctly identifies the signs of these quantities?

	$W_{\text{ext}}$	$W_{\text{elec}}$
A	Positive	Positive
B	Positive	Negative
C	Negative	Positive
D	Negative	Negative

- (A) A  
 (B) B  
 (C) C  
 (D) D

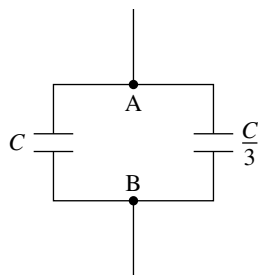


4. Three long wires, R, S, and T, are arranged in the  $xy$ -plane parallel to each other and the  $x$ -axis at the positions shown in the figure. The direction and current in each wire are indicated. Which of the following correctly indicates the magnitude and direction of the magnetic field at the origin of the coordinate system?
- (A)  $\frac{\mu_0 I}{\pi d}$  in the  $-z$ -direction
- (B)  $\frac{\mu_0 I}{\pi d}$  in the  $+z$ -direction
- (C)  $\frac{2\mu_0 I}{\pi d}$  in the  $-z$ -direction
- (D)  $\frac{2\mu_0 I}{\pi d}$  in the  $+z$ -direction

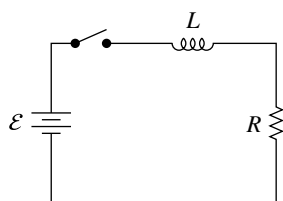


5. Points A and B are separated by a distance of 5 m and located in a uniform electric field of magnitude 1000 V/m directed in the  $+y$ -direction, as shown in the figure. The line between Point A and Point B makes an angle of  $37^\circ$  to the  $x$ -axis. When a proton (of charge  $+e$ ) is moved by an external force from Point A to Point B, what is the change in electric potential energy of the proton-field system?
- (A)  $-5000$  eV
- (B)  $-3000$  eV
- (C)  $+3000$  eV
- (D)  $+5000$  eV

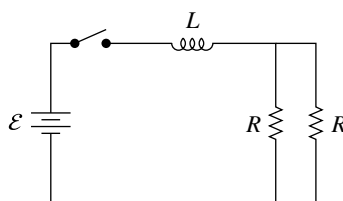




6. Two air-filled capacitors, with capacitance  $C$  and  $\frac{C}{3}$ , are connected in parallel, as shown. The equivalent capacitance between points A and B is  $C_{\text{eq}}$ . A dielectric is inserted into the capacitor with capacitance  $\frac{C}{3}$ , and the new equivalent capacitance between points A and B is  $2C_{\text{eq}}$ . What is the value of the dielectric constant of the dielectric?
- (A) 2  
 (B) 3  
 (C) 5  
 (D) 6



Circuit 1



Circuit 2

7. An inductor with inductance  $L$  is placed in series with a resistor of resistance  $R$ , a battery of emf  $\mathcal{E}$ , and a switch that is initially open, as shown in the Circuit 1 diagram. The switch is closed at time  $t = 0$ , and Circuit 1 reaches half of its steady-state current at  $t = t_1$ . Circuit 2 is constructed by adding a resistor of resistance  $R$  in parallel with the original resistor, as shown in the Circuit 2 diagram. The switch is closed at time  $t = 0$ , and Circuit 2 reaches half of its steady-state current at  $t = t_2$ . Which of the following correctly compares  $t_1$  and  $t_2$ , and provides a valid justification?
- (A)  $t_2 = t_1$ , because the time for an RL circuit to reach half of its steady-state current only depends on  $L$
- (B)  $t_2 = t_1$ , because both the time constant and the steady-state current double in Circuit 2 compared to Circuit 1
- (C)  $t_2 > t_1$ , because the equivalent resistance is greater in Circuit 2 than in Circuit 1, so the time constant is greater in Circuit 2
- (D)  $t_2 > t_1$ , because the equivalent resistance is less in Circuit 2 than in Circuit 1, so the time constant is greater in Circuit 2

**Questions 8 and 9 refer to the following.**

A sphere of uniform charge density has a positive net charge  $+Q$  and radius  $R_s$ . A small particle of positive charge  $+q$  can be moved through space both inside the sphere of uniform charge and outside the sphere.

8. The particle of charge  $+q$  is initially located a distance  $2R_s$  from the center of the sphere and then moved to the sphere's center. Which of the following correctly describes the change in the electric potential energy of the particle-sphere system,  $\Delta U_{ps}$ , when the particle is moved, and provides a valid justification?
- (A)  $\Delta U_{ps} > 0$ . The electric field due to the sphere's charge does positive work on the positively charged particle.
- (B)  $\Delta U_{ps} > 0$ . The motion of the positively charged particle is opposite to the direction of the electric field due to the sphere's charge.
- (C)  $\Delta U_{ps} = 0$ . The electric field due to the sphere's charge is zero at the center of the sphere.
- (D)  $\Delta U_{ps} = 0$ . As the positively charged particle is moved, positive work done outside the sphere is canceled by negative work of equal magnitude that is done inside the sphere.
9. With the particle held in place at the center of the charged spherical region, what is the magnitude of the net electric field due to both the particle and the sphere at a distance  $\frac{R_s}{2}$  from the sphere's center?

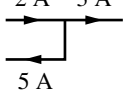
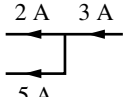
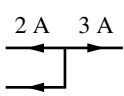
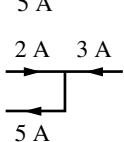
(A)  $\frac{1}{4\pi\epsilon_0(R_s/2)^2} \left( q + \frac{1}{8}Q \right)$

(B)  $\frac{1}{4\pi\epsilon_0(R_s/2)^2} \left( q - \frac{1}{8}Q \right)$

(C)  $\frac{1}{4\pi\epsilon_0(R_s/2)^2} \left( q + \frac{1}{2}Q \right)$

(D)  $\frac{1}{4\pi\epsilon_0(R_s/2)^2} \left( q - \frac{1}{2}Q \right)$

10. A junction in a circuit consists of three wires connected together. Which of the following diagrams could indicate the directions and magnitudes of the currents in the three wires?

- (A) 
- (B) 
- (C) 
- (D) 

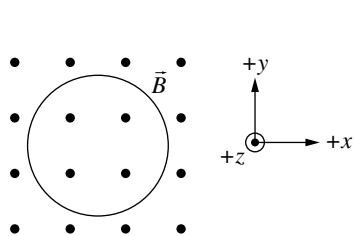


Figure 1

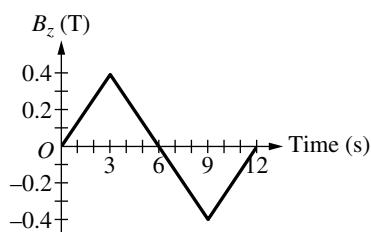
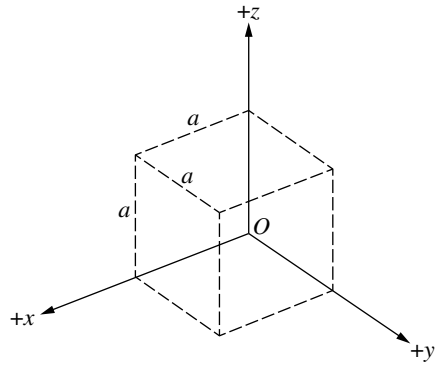


Figure 2

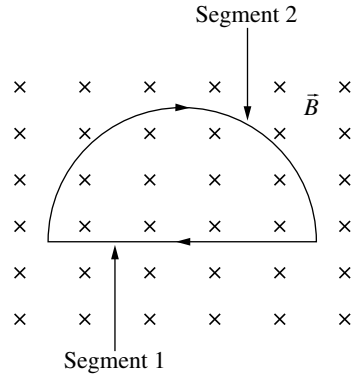
11. A metal wire of resistance  $10 \Omega$  is bent into a circular loop of radius  $0.10 \text{ m}$  and placed in a uniform magnetic field  $\vec{B}$ , as shown in Figure 1. The  $z$ -component  $B_z$  of the magnetic field is shown as a function of time in Figure 2, where positive indicates that the magnetic field is directed out of the plane of the figure. Which of the following correctly indicates the magnitude and direction of the current induced in the loop at time  $t = 6.0 \text{ s}$ ?

	Magnitude	Direction
A	3.8 mA	Clockwise
B	3.8 mA	Counterclockwise
C	0.42 mA	Clockwise
D	0.42 mA	Counterclockwise

- (A) A  
 (B) B  
 (C) C  
 (D) D



12. A Gaussian surface, in the shape of a cube of side length  $a$ , is oriented with one corner at the origin  $O$ , as shown in the figure. The cubical surface is in a region with charges (not shown) where the resulting electric field is described by  $\vec{E} = (bx)\hat{i}$ , where  $b$  is a positive constant. What is the total electric flux through the surface of the cube?
- (A) Zero  
(B)  $ba^3$   
(C)  $2ba^3$   
(D)  $6ba^3$



13. A semicircular loop with a clockwise current is placed in a uniform magnetic field that is directed into the page, as shown in the figure.  $\vec{F}_1$  is the net force on Segment 1, which is the straight portion of the loop.  $\vec{F}_2$  is the net force on Segment 2, which is the curved portion of the loop. Which of the following correctly indicates the directions and relative magnitudes of  $\vec{F}_1$  and  $\vec{F}_2$ ?

	Direction of $\vec{F}_1$	Direction of $\vec{F}_2$	Magnitudes
A	Toward the bottom of the figure	Toward the top of the figure	$F_1 = F_2$
B	Toward the bottom of the figure	Toward the top of the figure	$F_1 < F_2$
C	Toward the top of the figure	Toward the bottom of the figure	$F_1 = F_2$
D	Toward the top of the figure	Toward the bottom of the figure	$F_1 < F_2$

- (A) A  
(B) B  
(C) C  
(D) D

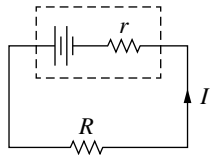


Figure 1

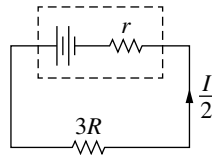
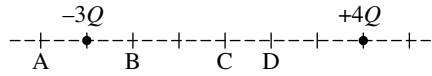


Figure 2

14. When a nonideal battery is connected to a resistor with resistance  $R$ , as shown in Figure 1, the current in the circuit is  $I$ . When the same nonideal battery is connected to a resistor with resistance  $3R$ , as shown in Figure 2, the current in the circuit is  $\frac{I}{2}$ . What is the internal resistance  $r$  of the battery?

- (A)  $\frac{R}{4}$   
(B)  $\frac{R}{2}$   
(C)  $R$   
(D)  $2R$



15. Two small spheres are arranged along a line and have charges of  $-3Q$  and  $+4Q$ , as shown in the figure. The tick marks on the axis are equally spaced. At which of the labeled tick marks does the electric field have the greatest magnitude?
- (A) A
  - (B) B
  - (C) C
  - (D) D

## Section II: Free-Response Questions

### FREE-RESPONSE QUESTION: MATHEMATICAL ROUTINES

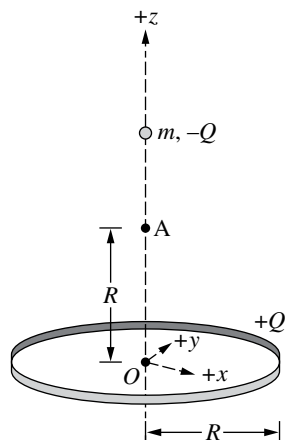


Figure 1

1. A thin, nonconducting ring is held fixed in the  $xy$ -plane with its center at the origin  $O$ . The ring has radius  $R$  and a positive charge  $+Q$  uniformly distributed around its circumference. A small sphere of mass  $m$  and negative charge  $-Q$  is on the  $z$ -axis, as shown in Figure 1. Point A is located on the  $z$ -axis at  $z = R$ .
- (a) i. In which of the following regions, if any, can the sphere be placed on the  $z$ -axis so that the net electric potential at Point A due to the sphere and the ring is zero? **Indicate** your answer by selecting one of the options below.

Above Point A only       Below Point A only

Either above or below Point A

There is no location the sphere can be placed so that the net electric potential is zero

Briefly **justify** your reasoning.

- ii. The sphere is released from rest on the  $z$ -axis some distance above the ring. All forces other than the electric force exerted by the ring are negligible. The sphere passes through the origin  $O$ , where it has speed  $v_O$ , and continues moving, eventually passing through the location  $z = -3R$ . **Derive** an expression for the speed of the sphere at the instant the sphere passes through the location  $z = -3R$ . Express your answer in terms of  $m$ ,  $Q$ ,  $R$ ,  $v_O$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference booklet.

The sphere is then removed. Figure 2 shows the ring and Point C, which is located at  $z = 2R$  on the  $z$ -axis.

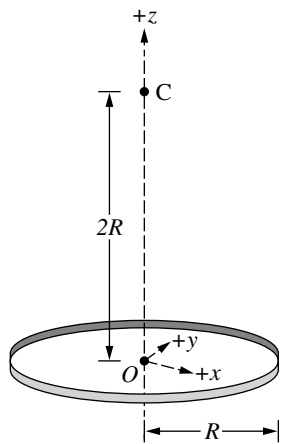


Figure 2

- (b) **Derive** an expression for the magnitude of the electric field due to the ring at Point C. Express your answer in terms of  $Q$ ,  $R$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference booklet.



## FREE-RESPONSE QUESTION: TRANSLATION BETWEEN REPRESENTATIONS

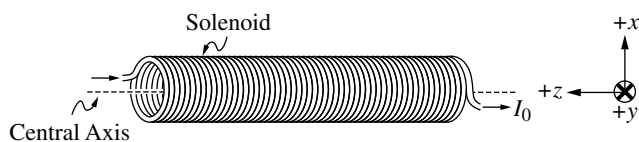


Figure 1: Side View

Note: Figure not drawn to scale

2. A long solenoid with its central axis along the  $z$ -axis has current  $I_0$ , as shown in Figure 1. The length of the solenoid is much larger than the radius of the solenoid loops.
- (a) Figure 2 shows a cross-sectional view of the middle of the solenoid, along with three square regions labeled 1, 2, and 3. Region 1 is centered on the central axis of the solenoid, Region 2 is inside the solenoid but offset from the central axis, and Region 3 is outside the solenoid.

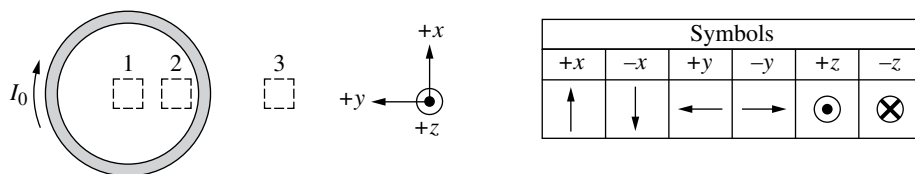


Figure 2: Cross-Sectional View

Inside the square for each region in Figure 2, **indicate** the direction of the magnetic field produced by the solenoid by drawing one of the symbols shown in the table. If the magnetic field is zero or negligible in any region, write “zero” just below that region.

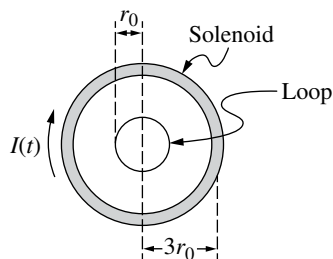


Figure 3

A small conducting loop of radius  $r_0$  is placed at the center of the solenoid, as shown in the cross-sectional view in Figure 3. The solenoid radius is  $3r_0$  and the solenoid has  $n_0$  turns per unit of length along the axis. Starting at time  $t=0$ , the current  $I$  in the solenoid is varied according to the equation  $I(t) = I_0(1 - bt^2)$ , where  $I_0$  and  $b$  are positive constants and positive current is taken to be in the clockwise direction.

- (b) **Derive** an expression for the absolute value of the induced emf in the smaller loop for times  $t \geq 0$ . Express your answer in terms of  $r_0$ ,  $n_0$ ,  $I_0$ ,  $b$ ,  $t$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference booklet.

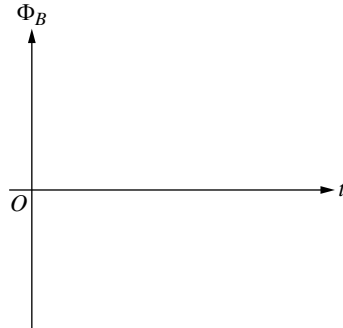


Figure 4

- (c) On the axes in Figure 4, **sketch** a graph of the magnetic flux  $\Phi_B$  through the small conducting loop due to the current in the solenoid as a function of time  $t$  from  $t=0$  until the flux reaches zero. On the horizontal axis of the graph, **label** the time at which the flux is zero with an expression in terms of the constant  $b$ .
- (d) **Indicate** the factor by which the vertical intercept of the graph you drew in part (c) would change, if at all, if the loop inside the solenoid had a radius of  $2r_0$ . Briefly **justify** your answer.

### FREE-RESPONSE QUESTION: EXPERIMENTAL DESIGN AND ANALYSIS

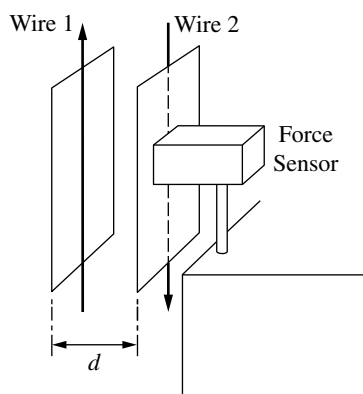


Figure 1

3. Students are investigating the relationship between the distance between current carrying wires and the magnitude of the magnetic force exerted between the wires using the setup shown in Figure 1. Two vertical wires, Wire 1 and Wire 2, are connected to power supplies (not shown) and carry currents in opposite directions. Each wire is attached to a board, and the board with Wire 2 is connected to a stationary force sensor. The boards are held parallel to each other and the distance  $d$  between the wires can be adjusted.

The students hypothesize that the force exerted by Wire 1 on Wire 2 is proportional to the inverse of the square of the distance between the wires.

- (a) **Describe** a procedure for collecting data that could be used to determine the relationship between the magnitude of the magnetic force exerted on the wires and the distance between the wires. Include any steps necessary to reduce experimental uncertainty.
- (b) **Describe** what quantities could be graphed and how the graph could be analyzed to test their hypothesis.

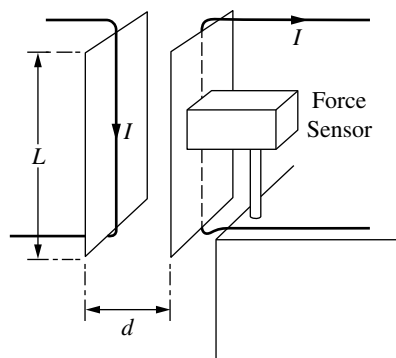


Figure 2

The students want to determine an experimental value for the magnetic permeability of free space,  $\mu_0$ . The same experimental setup is used and the current  $I$  is the same for both wires, as shown in Figure 2. The length  $L$  of the

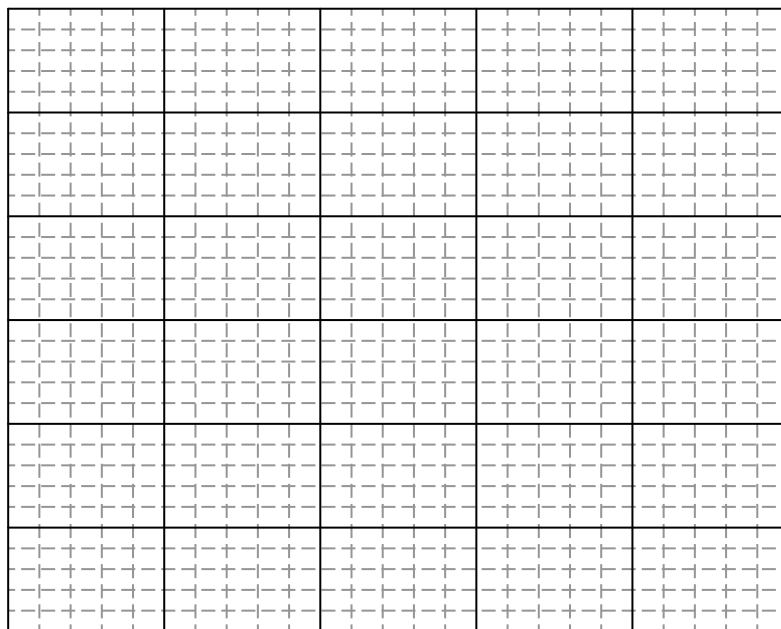
wire segments fixed to the boards are measured. In this experiment the distance  $d$  between the wires is held fixed and the force is measured while the current  $I$  is then varied. Using measured values of  $L=1.4$  m and  $d=0.005$  m, the students collect the data shown in the table.

$I$ (A)	$F$ (mN)		
2.8	0.35		
3.2	0.60		
3.6	0.65		
4.0	1.00		
4.4	1.10		

- (c) i. **Indicate** what measured and/or calculated quantities could be graphed to yield a straight line that could be used to calculate a numerical value for  $\mu_0$ . Use the blank columns in the table to list any calculated quantities you will graph other than the data provided.

Horizontal Axis: \_\_\_\_\_ Vertical Axis: \_\_\_\_\_

- ii. **Plot** the data points for the quantities indicated in part (c)(i) on the graph provided. Clearly scale and label all axes, including units, as appropriate.



- iii. **Draw** the best-fit line for the data graphed in part (c)(ii).
- (d) Using the line drawn in part (c)(iii) and the measured values  $L=1.4$  m and  $d=0.005$  m as needed, **calculate** the magnetic permeability of free space,  $\mu_0$ .

**FREE-RESPONSE QUESTION: QUALITATIVE/QUANTITATIVE TRANSLATION**

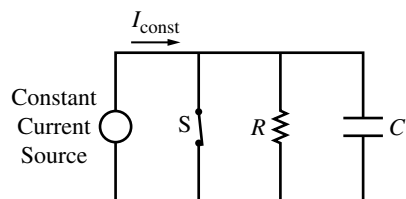


Figure 1

4. The figure shows a circuit consisting of a closed switch  $S$ , a resistor of resistance  $R$ , and an initially uncharged parallel-plate capacitor of capacitance  $C$  in parallel with an ideal constant current source that generates a constant current  $I_{\text{const}}$  in the direction indicated. At time  $t = 0$  the switch  $S$  is opened, and the current source continues generating a constant current  $I_{\text{const}}$ . The potential difference across the capacitor is taken to be positive when it has positive charge on its top plate.

(a) A long time after the switch has been opened, the rate of change  $\frac{d(\Delta V)}{dt}$  of

the potential difference across the capacitor approaches a constant value.

**Indicate** whether the value is positive, negative, or zero.

Positive     Negative     Zero

**Justify** your answer.

- (b) **Derive**, but do not solve, a differential equation for the rate of change

$\frac{d(\Delta V)}{dt}$  of the potential difference  $\Delta V$  across the capacitor after the switch

is opened. Express your answer in terms of  $R$ ,  $C$ ,  $\Delta V$ ,  $t$ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference booklet.

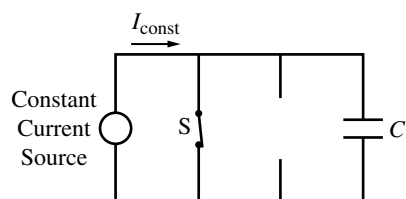


Figure 2

- (c) The process is repeated with the resistor removed from the circuit, leaving a gap in the middle branch, as shown in Figure 2. The switch is again opened with the capacitor initially uncharged, and the current source again generates a constant current  $I_{\text{const}}$ . **Indicate** whether the rate of change

$\frac{d(\Delta V)}{dt}$  of the potential difference across the capacitor shortly after (but

not immediately after) the switch is opened will be zero, a nonzero constant value, or a value with increasing magnitude.

Zero     Nonzero constant value     Value with increasing magnitude

**Justify** your answer by referring to the differential equation you wrote for part (b).

# Answer Key and Question Alignment to Course Framework

Multiple-Choice Question	Answer	Skill	Learning Objective	Essential Knowledge
1	D	3.B	13.2.A	13.2.A.1
2	A	2.B	11.4.A	11.4.A.1
3	B	3.B	8.1.A	8.1.A.3
4	C	2.A	12.4.A	12.4.A.1
5	B	2.B	9.2.B	9.2.B.2
6	C	2.A	10.4.A	10.4.A.5
7	D	3.C	13.5.A	13.5.A.3
8	B	3.c	9.3.A	9.3.A.1
9	A	2.A	8.6.A	8.6.A.1
10	D	3.B	11.7.A	11.7.A.2
11	D	2.B	13.2.A	13.2.A.2
12	B	2.A	8.5.A	8.5.A.2
13	A	2.C	12.3.B	12.3.B.1
14	C	2.D	11.5.B	11.5.B.2
15	B	2.C	8.3.A	8.3.A.2

Free-Response Question	Skill	Learning Objective
1	2.A, 3.B, 3.C	8.4.A, 9.2.A, 9.2.B, 9.3.A
2	1.A, 1.C, 2.A, 2.D, 3.B, 3.C,	12.4.A, 13.1.A, 13.2.A
3	1.B, 2.B, 2.D, 3.A	11.3.B, 12.3.B, 12.4.A
4	2.A, 2.D, 3.B, 3.C	10.3.A, 11.1.A, 11.7.A, 11.3.B, 11.8.B

THIS PAGE IS INTENTIONALLY LEFT BLANK.



# Scoring Guidelines

## Scoring Guidelines for Question 1: Mathematical Routines

**10 points****Learning Objectives:** 8.4.A 9.2.A 9.2.B 9.3.A

(a) i For selecting “Either above or below Point A”, with an attempt at a relevant justification **1 point**

For indicating that because the ring and the sphere have opposite sign charges, the net electric potential only depends on the relative distances of the ring and sphere from Point A **1 point**

**Example Response:***Either above or below Point A*

*The net potential at Point A is the sum of the potentials of the ring and the sphere. Because potential is a scalar quantity, and because the ring and the sphere have opposite signs, there are two locations (above and below Point A) where the potential due to the sphere is equal and opposite in sign to the potential of the ring.*

(a) ii For a multi-step derivation starting with a statement of conservation of energy (e.g.,  $E_{\text{total}} = U + K = \text{constant}$ ) **1 point**

For correct expressions for the electric potential energy at two different locations **1 point**

For using the correct locations, at both  $z = 0$  and  $z = -3R$ , for the potential energy expressions **1 point**

For substituting correct expressions for the kinetic energy at  $z = 0$  and  $z = -3R_0$  **1 point**

For a correct answer,  $\sqrt{v_o^2 - \frac{2kQ^2}{mR} \left(1 - \frac{1}{\sqrt{10}}\right)}$ , or equivalent. **1 point**

**Example Response:**

$$E_{\text{total}} = U + K = \text{constant}$$

$$[U + K]_{z=0} = [U + K]_{z=-3R_0}$$

$$-k \frac{Q^2}{R} + \frac{1}{2} m v_o^2 = -\frac{kQ^2}{\sqrt{(3R)^2 + R^2}} + \frac{1}{2} m v^2$$

$$-k \frac{Q^2}{R} + \frac{1}{2} m v_o^2 = -\frac{kQ^2}{\sqrt{10}R^2} + \frac{1}{2} m v^2$$

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_o^2 - k \frac{Q^2}{R} + \frac{kQ^2}{\sqrt{10}R} = \frac{1}{2} m v_o^2 - \frac{kQ^2}{R} \left(1 - \frac{1}{\sqrt{10}}\right)$$

$$v^2 = v_o^2 - \frac{2kQ^2}{mR} \left(1 - \frac{1}{\sqrt{10}}\right)$$

$$v = \sqrt{v_o^2 - \frac{2kQ^2}{mR} \left(1 - \frac{1}{\sqrt{10}}\right)}$$

**Total for Part (a)****7 points**



---

(b) For a multi-step derivation starting with an equation for the electric field of a charge distribution **1 point**

---

For an indication that only the  $z$ -component of the field need be considered (e.g., by including a  $\cos\theta$  term in the integral) **1 point**

---

For a correct final answer **1 point**

**Scoring note:** The answer can be in terms of either  $\epsilon_0$  or  $k$ .

**Example Response**

$$\begin{aligned}\vec{E} &= k \int \frac{dq}{r^2} \hat{r} \\ E = E_z &= k \int \frac{dq}{r^2} \cos\theta = k \int \left( \frac{dq}{(2R)^2 + R^2} \right) \left( \frac{2R_0}{\sqrt{R^2 + 4R^2}} \right) \\ E = E_z &= k \left( \frac{1}{5R^2} \right) \left( \frac{2}{\sqrt{5}} \right) \int dq \\ &= k \left( \frac{2Q}{5\sqrt{5}R^2} \right)\end{aligned}$$

---

(b) **Alternate Solution** **1 point**

For a multi-step derivation that relates the electric field to the derivative of the electric potential.

---

For a correct potential expression as a function of  $z$  **1 point**

---

For correctly differentiating the electric potential expression, substituting  $z = 2R_0$  into the derivative, and getting a final answer that is consistent with the electric potential expression found **1 point**

**Scoring note:** The answer can be in terms of either  $\epsilon_0$  or  $k$ .

**Example Response (Alternate Solution):**

$$\begin{aligned}E = E_z &= -\frac{dV}{dz} \\ V &= k \int \frac{dq}{r} = k \frac{\int dq}{\sqrt{R^2 + z^2}} = k \frac{Q}{\sqrt{R^2 + z^2}} = kQ(R^2 + z^2)^{-1/2} \\ \frac{dV}{dz} &= kQ \left( -\frac{1}{2} \right) (R^2 + z^2)^{-3/2} (2z) = -kQ \frac{z}{(R^2 + z^2)^{3/2}} \\ z &= 2R: \\ |E| = |E_z| &= \left| -\frac{dV}{dz} \right| = kQ \frac{2R}{(R^2 + (2R)^2)^{3/2}} = kQ \frac{2R}{(5R^2)^{3/2}} \\ |E| &= k \left( \frac{2Q}{5\sqrt{5}R^2} \right)\end{aligned}$$

---

**Total for part (b)** **3 points**

**Total for question 1** **10 points**

---

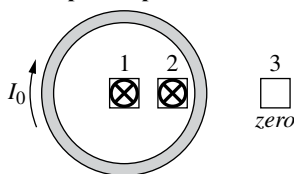
## Scoring Guidelines for Question 2: Translation Between Representations

12 points

**Learning Objectives:** 12.4.A 13.1.A 13.2.A

- (a) For indicating consistent fields along the  $z$ -axis in Regions 1 and 2 1 point  
**Scoring note:** Consistent fields include the following:  
 A cross within a circle for both regions  
 A dot within a circle for both regions
- 
- For indicating that for Region 1 the field is into the plane of the figure 1 point
- 
- For writing “zero” next to Region 3 1 point

**Example Response:**



**Total for Part (a)** 3 points

- (b) For a multistep derivation that starts with  $\mathcal{E} = -\frac{d\Phi}{dt}$  1 point
- 
- For substituting the small loop's area into an expression for the magnetic flux 1 point
- 
- For substituting a correct expression for the magnitude of the magnetic field inside a solenoid 1 point
- 
- For correctly taking the time derivative of an expression for either the magnetic flux, the magnetic field, or the current 1 point
- Scoring note:** The expression for flux, field, or current need not be correct to earn this point.

**Example Response:**

$$A = \pi r_0^2 \text{ and } B = \mu_0 n_0 I$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = \left| \frac{d(\vec{B} \cdot \vec{A})}{dt} \right| = A \left| \frac{dB}{dt} \right|$$

$$|\mathcal{E}| = (\pi r_0^2) \left( \mu_0 n_0 \left| \frac{dI}{dt} \right| \right) = \pi r_0^2 \mu_0 n_0 |-I_0 (2bt)|$$

$$|\mathcal{E}| = 2bI_0 \pi r_0^2 \mu_0 n_0 t$$

**Total for Part (b)** 4 points

---

(c) For drawing a curve that is concave-downward everywhere **1 point**

---

For a line or curve that contains the point  $\left(\frac{1}{\sqrt{b}}, 0\right)$  on the horizontal axis, with  $\frac{1}{\sqrt{b}}$  clearly indicated **1 point**

**Scoring note:** The curve need not extend to include negative flux values for times  $t > \frac{1}{\sqrt{b}}$  to earn this point.

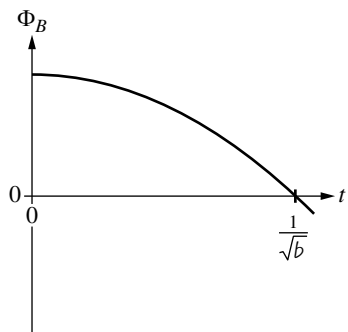
---

For a continuous curve that has a positive maximum on the vertical axis and zero slope at the maximum **1 point**

**Scoring note:** Drawing the mirror image across the horizontal axis earns full credit.

---

**Example Response**



**Total for Part (c) 3 points**

---

(d) For indicating that the intercept will increase, with an attempt at a justification using functional dependence **1 point**

---

For indicating that the flux (or area) is proportional to the square of the loop's radius so will increase by a factor of 4 **1 point**

---

**Example Response**

*The vertical intercept will increase by a factor of 4. This is because the loop area is proportional to  $r^2$ , and if the radius doubles, the area and therefore the flux will increase by a factor of 4.*

**Total for part (d) 2 points**

---

**Total for question 2 12 points**

## Scoring Guidelines for Question 3: Experimental Design and Analysis 10 points

**Learning Objectives:** 11.3.B 12.3.B 12.4.A

(a) For indicating that current is held constant, distance between the wires is varied, and force is measured **1 point**

For indicating that measurements should be taken for several wire distances **1 point**

**Example Response:**

*Use the power supply to create constant currents in the wires. Measure the plate separation  $d$ . Record the force reading  $F$  on the force sensor. Repeat this for a total of 10 values of plate separation distances, using the same wire currents each time.*

**Total for Part (a)** **2 points**

(b) For indicating that a graph involving force and distance can be used to determine the relationship **1 point**

For describing a valid method of determining whether the graph is consistent with the hypothesis **1 point**

**Example Response:**

*Plot  $F$  vs  $\frac{1}{d^2}$ . If it is linear, the hypothesis is correct, if not, it's incorrect.*

**OR**

*Plot  $F$  vs  $\frac{1}{d}$ . If it is linear, then the hypothesis is incorrect.*

**OR**

*Plot  $F$  vs  $d$ , use power regression to find functional dependence.*

**Total for Part (b)** **2 points**

(c) i For indicating appropriate quantities that would produce a straight-line graph to determine  $\mu_0$  **1 point**

**Examples:**

Horizontal Axis:  $I^2$

Vertical Axis:  $F$

**OR**

Horizontal Axis:  $I$

Vertical Axis:  $\sqrt{F}$

(c) ii For labeling the axes correctly (including units) with a linear scale, where the data uses at least half the range on each axis of the graph **1 point**

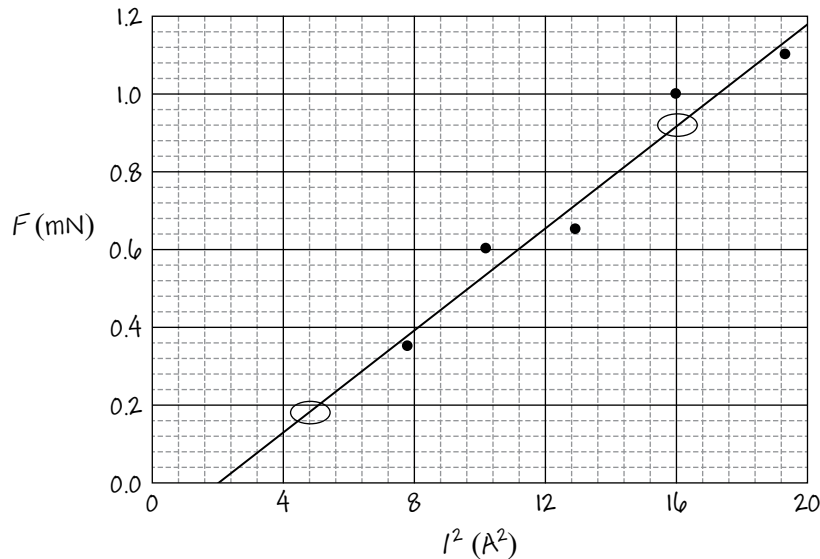
For plotting the data points correctly **1 point**

(c) iii For drawing a straight line that approximates the trend of the data **1 point**

**Example Response for (c)(i), (c)(ii), (c)(iii) combined:**

Horizontal Axis:  $I^2$       Vertical Axis:  $F$

$I$ (A)	$F$ (mN)	$I^2$ (A <sup>2</sup> )	
2.8	0.35	7.8	
3.2	0.60	10.2	
3.6	0.65	13.0	
4.0	1.00	16.0	
4.4	1.10	19.4	



**Scoring note:** The circled points are used for the slope calculation in part (d). It is not necessary to indicate them here to earn credit in part (c).

**Total for part (c)**      **4 points**

**(d)** For using an appropriate feature of the best-fit line to calculate  $\mu_0$       **1 point**

For correctly relating the slope to  $\mu_0$       **1 point**

**Scoring note:** Linear regression on the given data, graphed as  $F$  vs  $I^2$ , yields a slope of  $0.0657 \text{ mN/A}^2$  and  $\mu_0 = 1.47 \times 10^{-6} \text{ N/A}^2$ .

**Example Response:**

The line contains the points  $(4.8 \text{ A}^2, 0.18 \text{ mN})$  and  $(16.0 \text{ A}^2, 0.92 \text{ mN})$ .

The slope of the line is

$$\text{slope} = \frac{0.92 - 0.18}{16.0 - 4.8} \text{ mN/A}^2 = \frac{0.74}{11.2} \text{ mN/A}^2 = 0.0661 \text{ mN/A}^2 \text{ or } 0.0661 \times 10^{-3} \text{ N/A}^2$$

Force and current are related as follows.

$$F = ILB \quad \text{and} \quad B = \frac{\mu_0 I}{2\pi d}$$

$$F = \frac{\mu_0}{2\pi} I^2 \frac{L}{d}$$

$$\text{So the slope of the graph is } \text{slope} = \frac{\mu_0 L}{2\pi d} \quad \text{and} \quad \mu_0 = 2\pi (\text{slope}) \frac{d}{L}$$

$$\mu_0 = 2\pi (0.0661 \times 10^{-3} \text{ N/A}^2) \frac{0.005 \text{ m}}{1.4 \text{ m}} = 1.48 \times 10^{-6} \text{ N/A}^2$$

**Total for part (d)**      **2 points**

**Total for question 3**      **10 points**

## Scoring Guidelines for Question 4: Qualitative/Quantitative Translation

8 points

**Learning Objectives:** **10.3.A** **11.1.A** **11.7.A** **11.3.B** **11.8.B**

- |     |  |         |
|-----|--|---------|
| (a) | For selecting “Zero,” with an attempt at a justification                             | 1 point |
|     | For indicating that the charge builds up to a maximum amount and no longer changes   | 1 point |
|     | For indicating that a constant charge corresponds to a constant potential difference | 1 point |

**Example Response:**

Zero.

*After the switch is closed, charge will start to build up on the capacitor and it will eventually stop charging. Potential difference is proportional to capacitor charge, so the potential difference will not be changing after a long time.*

**Total for Part (a)**      **3 points**

- |     |  |         |
|-----|--|---------|
| (b) | For a multi-step derivation starting with a correct application of either Kirchhoff’s junction rule or loop rule (e.g., $I_{\text{const}} = I_R + I_C$ or $\Delta V_C - \Delta V_R = 0$ )                        | 1 point |
|     | For substituting correct expressions of the current in the resistor, $\frac{\Delta V}{R}$ , and the current in the capacitor branch, $\frac{dq}{dt}$ , into an equation that expresses Kirchhoff’s junction rule | 1 point |
|     | For correctly substituting the current in the capacitor branch as $\frac{d(C\Delta V)}{dt}$  | 1 point |

**Example Response (Starting with loop rule):**

$$\Delta V_C - \Delta V_R = 0$$

$$\Delta V_C = \Delta V_R = I_R R$$

$$I_{\text{const}} = I_R + I_C$$

$$\Delta V_C = (I_{\text{const}} - I_C) R = \Delta V_C = \left( I_{\text{const}} - \frac{dq}{dt} \right) R$$

$$\Delta V_C = \left( I_{\text{const}} - \frac{d(C\Delta V)}{dt} \right) R$$

$$\frac{d(\Delta V)}{dt} = \frac{I_{\text{const}}}{C} - \frac{\Delta V_C}{RC}$$

**Example response (Starting with junction rule):**

$$I_{\text{const}} = I_R + I_C = \frac{\Delta V}{R} + \frac{dq}{dt}$$

$$I_{\text{const}} = \frac{\Delta V}{R} + \frac{d(C\Delta V)}{dt}$$

$$C \frac{d(\Delta V)}{dt} = I_{\text{const}} - \frac{\Delta V}{R}$$

$$\frac{d(\Delta V)}{dt} = \frac{I_{\text{const}}}{C} - \frac{\Delta V_C}{RC}$$

**Total for Part (b)**      **3 points**

- 
- (c) For attempting to use functional dependence in the differential equation in part (b) by relating resistance to either a relevant term in the equation or to the derivative of the potential difference across the capacitor **1 point**
- Scoring note:** To earn this point it is not necessary to indicate that the resistor is removed or the resistance becomes infinite, and it is not necessary to use the functional dependence correctly.
- 

For a selection and an explanation that are both consistent with the equation from part (b), and an explanation in which the resistance is infinite or approaches infinity **1 point**

---

**Example Response:**

*Nonzero constant value.*

*If the resistor were not present, that is the same as letting  $R$  go to infinity. In the differential equation,*

*this means that the term  $\frac{\Delta V}{R}$  goes to zero and can be ignored. The result is  $\frac{d(\Delta V)}{dt} = \frac{I_0}{C}$ , which is a positive constant.*

---

**Total for part (c)** **2 points**

---

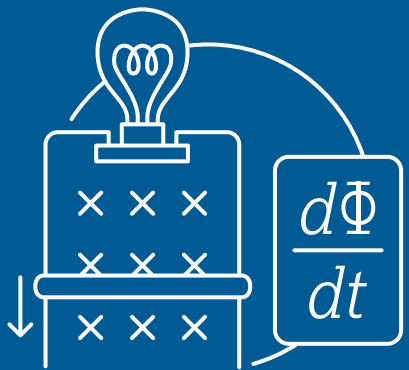
**Total for question 4** **8 points**

**AP PHYSICS C: ELECTRICITY AND MAGNETISM**

---

# Appendix





# Vocabulary and Definitions of Important Ideas in AP Physics

---

## Vocabulary and Word Choice in AP Physics

The discussions below are included to elaborate on the choice of words and vocabulary used within AP Physics. Many of these words, when used in the context of AP Physics, have very specific and intentional meanings. The intentional use of certain words within specific contexts can have a significant impact on student understanding as well as their ability to communicate that understanding with others.

The AP Physics Exam will NOT directly assess student understanding of physics vocabulary. For instance, students will not be asked to identify the correct definition of acceleration, or the difference between a system and an object. Students will be expected to use these definitions in contextually appropriate situations. Descriptions and definitions, found in the appendix, are intended to be used as starting points for discussions and are included to help students be more conscientious about the language they use to describe a scenario—that they are inherently thinking more about the underlying principles and ideas that apply to that scenario. This can lead to a deeper and more robust understanding of the course content.

Often in physics contexts, words have specific meanings and are used differently than in colloquial conversation. Common examples of words that have specific physics definitions include object, system, momentum, and work. Students need to be aware of these subtleties, so that they can communicate appropriately about the details of a scenario. Ultimately, the language used to describe the natural world can be very important when helping students build understanding of physics concepts in measured and intentional increments.

## Models

### **A PHYSICAL, MATHEMATICAL, OR CONCEPTUAL REPRESENTATION OF A SYSTEM OF IDEAS, EVENTS, OR PROCESSES.**

A scientific model can be thought of as the set of rules that describe a physical phenomenon. The model sets out the boundaries within which the scientist will consider

that phenomenon. Typically, these boundaries simplify a complex scenario to make the analysis and description of that scenario easier and more accessible, particularly to students just beginning their studies in physics.

Consider perhaps the most common example of an introductory physics problem: “A block is at rest at the top of an inclined plane. Determine the speed of a block at the bottom of the inclined plane.” What should students consider in their calculations? What model of the block/incline/earth should students use to describe this situation? To describe the subtleties of the scenario, students would need to consider friction and air resistance; the slight increase in gravitational field as the block slides downward; and the loss of energy to vibrations, sound, and thermal energy. Does the coefficient of friction decrease slightly as the abrasion between the block and incline subtly smooths the surfaces? Should the density, temperature, and relative humidity of the air be considered? Clearly, the physics of even such a straightforward example of a block-on-a-ramp can raise many questions to be considered.

Therefore, in introductory physics courses, phenomena are typically analyzed in the most basic conditions, using the most simplified models. This allows students to focus on big concepts and ideas, before exploring more complex models that include more detailed considerations. For example, when modeling Earth, we typically consider it to be uniform density, spherical, and an inertial frame of reference, even though none of those properties are completely accurate. Most often, only gravitational effects from the Sun are considered. Even tidal effects from the Moon are only considered after introductory courses. This spherical, uniform description of Earth is a simplified model that is used to focus on bigger concepts without getting stuck with extraneous details and nuance. In the earlier block-on-a-ramp example, virtually all of the effects listed are considered negligible and are ignored in favor of obtaining an answer that is within the level of accuracy needed for the course. The mathematics required to describe these effects tends to get complex quickly. It is important, however, that students understand they are using a simplified model so that later extensions can be added in the context of refining the model—a normal scientific process.

The models chosen to simplify the universe have been done so with alignment to their respective AP Physics courses. These models are elaborated on within the boundary statements provided in the course frameworks, as well as in the conventions for the AP Exams, listed on the equation sheets. While nuances of these models are described in detail within each course's course framework, these models can be summarized as follows.

Unless otherwise stated, students may assume that:

- Frames of reference are inertial.
- Air resistance is negligible.
- Frictional/drag forces are negligible.
- Edge effects of charged plates are negligible.
- Strings, springs, and pulleys are ideal.

## Representations

### A METHOD OF UNDERSTANDING AND COMMUNICATING UNDERSTANDINGS ABOUT PHYSICS.

Once deciding on the boundaries of a model, scientists must decide how to communicate those boundaries to others. A representation is a depiction of a model or aspects of that model. Representations can take many forms, and scientists are consistently developing new representations.

Representations that are frequently used within AP Physics C: Electricity and Magnetism include (but are not limited to):

- Written descriptions
- Drawings and pictures
- Diagrams or schematics
- Mathematical equations or sets of equations
- Graphs and data tables
- Charts
- Motion maps
- Energy bar charts
- Momentum charts
- Free-body diagrams
- Force diagrams
- Vector maps
- Field lines or field line maps
- Equipotential lines

Students will benefit from familiarity with as many different representations as possible. What makes a concept or idea clear to one student using one representation may not be clear to another. The more methods that students are given to access

and describe content, the more likely they are to use those descriptions. The depth to which a student understands course content is related to the variety of representations with which that student can communicate their knowledge. True understanding is demonstrated through the ability to use many different representations in many different situations. To this end, the AP Physics C: Electricity and Magnetism Exam will use many representations, as well as require students to create many representations.

## Objects

### A PHYSICAL THING WHERE THE INTERNAL STRUCTURE AND PROPERTIES OF THE THING ARE IGNORED.

Whether it be a cow, the Earth, a car, or pencil, the object model of these entities has a very specific meaning. Within the context of AP Physics, using the word "object" denotes some key characteristics, and is used, as most models are, to simplify the analysis and descriptions of the interactions between two or more masses. Most notably, an object has no internal structure or surface properties. An object can be considered as a collection of atoms or molecules that stick together in a functional way. A person could imagine handling an object, picking it up, as though the object had no internal structure.

Consider a truck. Most often, it's simply "a truck." The user of the truck does not consider the multitude of components that make a truck a truck: the engine, the doors and windows, the wheels, the frame, the radio, the suspension, and so on. The user treats the truck as a single object, neglecting the constituent parts and structure of the truck itself.

Similar to a well-packed box, an object is treated the same from different perspectives. The truck is a truck if viewed from the top, bottom, or side. However, when carrying a load of unsecured bricks, the object model of the truck may not be sufficient. The motion of the bricks within the truck may affect the behavior of the truck itself. Sudden accelerations—in any direction—may have significant effects on how the truck behaves.

Furthermore, using the object model ignores the physical size of the object itself. Objects cannot be compressed, twisted, or rotated because the physical dimensions of the object are ignored. When considering a truck as an object, there would be no need to make the distinction between the front, back, or sides of the truck. However, in the physical world, pushing the top of a truck has a different effect than pushing the bottom or middle of the truck. If the truck is modeled as an object, these effects are ignored; the location of the application of the force is not considered.

A notable discussion of some nuances of the object model can be found when analyzing friction. Friction, by definition, is the interaction of two objects in physical contact with each other. The amount of friction is inherently tied to the structure and properties of those objects. For instance, two wooden blocks will slide across each other differently if they are covered with sandpaper than if they are covered in grease. The surfaces of the blocks matter when it comes to describing their interactions. However, the blocks may still be treated as objects because the force of friction exerted on one block by the other block does not depend on the size or shape of the blocks. The amount of area of the blocks that are in contact with each other does not change the force of friction, and so the blocks may still be modeled as objects.

The object model is used throughout AP Physics C: Electricity and Magnetism to simplify the analysis of most phenomena. An “object” can be anything because what the object is is not important to the analysis. The properties that matter to the analysis—the mass of the object, the coefficient of friction between the object and a surface, the speed of the object, and so on—can be used to describe any number of physical things. In this case, it is up to the student to create their own mental representation of the situation.

## Systems

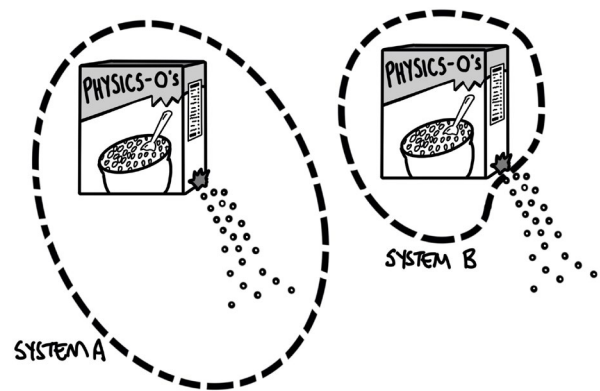
### A COLLECTION OF OBJECTS THAT ARE ANALYZED TOGETHER.

A system is how a physicist chooses to group objects together to analyze a given scenario. As students gain a deeper understanding of physics concepts, they will notice that how systems are chosen can significantly simplify (or significantly complicate) the analysis of a problem. There is no single right or wrong way to group objects, but often the preferred method is to choose a system that simplifies the analysis.

Note that in special cases, a system can itself be reduced to a single object. This can happen when the behaviors and interactions of the individual parts of the system do not affect the behavior or analysis of the system as a whole. Consider a box of cereal. In reality, this is a very complex system. A cardboard box, a plastic bag, and a large number of pieces of cereal within. However, the complex motion of the pieces of cereal within the box are not important to consider when handling the box of cereal. Therefore, the entire box–bag–cereal system can itself be considered as a single object.

Unless the system is chosen to be the entire observable universe (which would require an exceedingly complex analysis), a small group of objects chosen to be a

system will exist as part of a larger local environment. The system as a whole then may interact with that environment. Consider the box of cereal above. Perhaps the box is torn, and cereal begins to spill. The physicist has a decision to make with regard to their system: continue to include every piece of cereal as part of the system (System A) or consider only the cereal inside the box to be part of the system (System B). See the figure below.



Analyzing System A will be exceedingly complex, as the small pieces of cereal move, bounce, accelerate, and collide with each other and the environment and scatter. Analyzing System B is much simpler: the box is losing mass to the environment, but the box–bag–cereal system may be modeled as a single object that has a changing mass.

### Constant or conserved?

The cereal example is a good place to discuss the subtlety between the terms *constant* and *conserved*, and how the choice of the system determines whether a quantity is constant or conserved. For the leaking cereal box, the total amount of cereal is *conserved*, in both System A and System B. In both choices of system, the total amount of cereal that exists does not change. However, the choice of system does influence whether the amount of cereal within that system is *constant*. In the first choice, where the student decides to continue to include each individual piece of cereal as part of the system, even as the cereal spills from the box, the total amount of cereal within the system is constant. In the second choice, where the student decides to only consider the cereal within the box as part of the system, the total amount of cereal within the system decreases, and is not constant. However, this cereal is still conserved—the cereal does not simply vanish, disappear, or cease to exist because it is not selected to be part of the system. The cereal is transferred out of the system.

Suppose students wanted to analyze the energy of the box of cereal as it falls toward Earth. In the box–Earth system, total mechanical energy is both conserved and constant. The total amount of energy within the system does not change as the box gains kinetic energy, and the gravitational potential energy of the box–Earth system decreases. However, in a system consisting only of the box, the total amount of energy that system is not constant, but energy is conserved. The kinetic energy of the box increases, but this increase in energy is due to the transfer of energy into the box system by the external force of gravity doing work on the box. The energy transferred into the box by the force of gravity is not “new” energy that was created by gravity—the total energy of the universe has remained the same and has been conserved.

## Particle vs. Point Mass vs. Point Particle vs. Point Charge

In the search to make simplified models of the universe, physicists have developed a wide variety of terms. All of these terms serve the same purpose: to alert others to the simplifications and assumptions made by the model used during the analysis. However, over time, the meanings of these terms drift—or other words that are more favorable replace them. While the course framework for AP Physics 2 outlines the expectations of terms to teachers and students, it is in the interest of students to be aware of additional language used by physicists in a wide variety of settings. In the most traditional interpretation, a *particle* is an elementary piece of matter that is indivisible and has no internal structure. A particle evokes a mental model of a small piece of matter that interacts with other particles via collisions, like millions of tiny billiard balls on a table. Originally, protons and neutrons and electrons were considered fundamental particles that were indivisible. However, as our understanding of the nature of matter has developed, so has our understanding of these particles. Considering protons and neutrons as particles became increasingly problematic as their properties were studied. As physicists continued to explore the nature of the small, it was discovered that protons and neutrons do have structure and are made up of quarks. These quarks themselves have properties as well. Physicists then started to classify particles as bosons and fermions and hadrons. The more that was learned about these particles, the less appropriate the term *particle* became in order to evoke the mental model of these behaviors and properties. Even so, the term has stuck, and *particle* is still consistently used by many physicists when referring to subatomic pieces of matter, particularly when ignoring the wave nature and quantum properties of that matter.

The modifier *point* preceding mass, *particle*, or *charge* typically indicates to the reader that all other properties are ignored. For instance, a point mass ignores the size, shape, and distribution of that mass, and typically indicates a neutral charge. Students often struggle with understanding these terms, as they are incredibly abstract representations of the universe. It has been found that students have an easier time learning and applying concepts when using the object model, as objects provide a more concrete foundation upon which students may apply their simplifications. Similar to *particle*, the use of the terms *point mass* and *point charge* (point particle less so) continues in many physics classrooms, and so it is prudent for physics students to be aware of their meaning.

A *point charge* can be introduced as another important simplifying model in physics, like object. A point charge is a charged system that can be modeled as an object because it has a size that is very small compared to the separation of the object or system from other distances in the situation being analyzed. As a result, the internal structure of a point charge can be ignored. Its charge (and potentially mass) will be the only properties of interest. Charge itself is a word that should be used in specific circumstances. Charge is a property of an object, and not an object itself. The amount and type of charge can be specified. Consider the sentence, “A charge is moving between two horizontal, charged, conducting plates.” What is the charge? Is it an electron? If so, electrons have wave properties, which will probably be ignored. Is the charge on a small sphere? Is the sphere conducting or nonconducting? That matters, because polarization of the sphere between the plates will affect the motion of the sphere. Is the charge a point charge? If so, this implies that the effects of its size and structure are negligible. If this model has not been carefully developed, it may be confusing to have an artificial, abstract point charge that does not exist in nature interact with the very real and plausible concrete scenario of two plates. Within the context of physics, the use of the term *object* simplifies many of these questions. An *object* with charge  $+q$ , moves between two horizontal, charged conducting plates. The object has mass, and charge, but cannot be polarized because it has no physical size. Referring to an object clearly ignores the wave-particle duality that may confuse the use of an electron. Using *object* has the advantage of clearly denoting which properties are relevant, while also anchoring this mental abstraction to a physical, tangible piece of matter.

**AP PHYSICS C: ELECTRICITY  
AND MAGNETISM**

**Table of  
Information:  
Equations**

## ADVANCED PLACEMENT PHYSICS ELECTRICITY AND MAGNETISM TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Coulomb constant,	$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} (\text{T} \cdot \text{m}) / \text{A}$
Proton mass,	$m_p = 1.67 \times 10^{-27} \text{ kg}$
Neutron mass,	$m_n = 1.67 \times 10^{-27} \text{ kg}$
Electron mass,	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Elementary charge,	$e = 1.60 \times 10^{-19} \text{ C}$
1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
Speed of light,	$c = 3.00 \times 10^8 \text{ m/s}$
1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Universal gravitational constant, $G = 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg} \cdot \text{s}^2) = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$	
Magnitude of the acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$	
Magnitude of the gravitational field strength at Earth's surface, $g = 9.8 \text{ N/kg}$	

UNIT SYMBOLS	
ampere,	A
coulomb,	C
electron volt,	eV
farad,	F
henry,	H
hertz,	Hz
joule,	J
kilogram,	kg
meter,	m
newton,	N
ohm,	$\Omega$
second,	s
tesla,	T
volt,	V
watt,	W

PREFIXES		
Factor	Prefix	Symbol
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^{-2}$	centi	c
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n
$10^{-12}$	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
$\theta$	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan \theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	$\infty$

The following conventions are used in this exam:

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.
- The electric potential is zero at an infinite distance from an isolated point charge.
- The direction of current is the direction in which positive charges would drift.
- All batteries, wires, and meters are assumed to be ideal unless otherwise stated.

ELECTRICITY AND MAGNETISM

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2} = k \frac{|q_1q_2|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{total}} = \int \rho(r) dV$$

$$U_E = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$\Delta V = - \int_a^b \vec{E} \cdot d\vec{r}$$

$$E_x = - \frac{dV}{dx}$$

$$\Delta U_E = q\Delta V$$

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$U_C = \frac{1}{2} Q\Delta V$$

$$\kappa = \frac{\epsilon}{\epsilon_0}$$

$$I = \frac{dq}{dt}$$

$$I = \int \vec{J} \cdot d\vec{A}$$

$$\vec{E} = \rho\vec{J}$$

$$R = \frac{\rho\ell}{A}$$

$$I = \frac{\Delta V}{R}$$

$$P = I\Delta V$$

*A* = area  
*C* = capacitance  
*d* = distance  
*E* = electric field  
*F* = force  
*I* = current  
*J* = current density  
*ℓ* = length  
*P* = power  
*q* = charge  
*Q* = charge  
*r* = radius, distance, or position  
*R* = resistance  
*t* = time  
*U* = potential energy  
*V* = electric potential or volume  
*ε* = electric permittivity  
*ρ* = resistivity or charge density  
*κ* = dielectric constant  
*Φ* = flux

$$R_{\text{eq},s} = \sum_i R_i$$

$$\frac{1}{R_{\text{eq},p}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{C_{\text{eq},s}} = \sum_i \frac{1}{C_i}$$

$$C_{\text{eq},p} = \sum_i C_i$$

$$\tau = R_{\text{eq}} C_{\text{eq}}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{\ell} \times \hat{r})}{r^2}$$

$$\vec{F}_B = \int I(d\vec{\ell} \times \vec{B})$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$B_{\text{sol}} = \mu_0 nI$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = - \frac{d\Phi_B}{dt}$$

$$|\mathcal{E}_{\text{sol}}| = N \left| \frac{d\Phi_B}{dt} \right|$$

$$L_{\text{sol}} = \frac{\mu_{\text{core}} N^2 A}{\ell}$$

$$U_L = \frac{1}{2} LI^2$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$\tau = \frac{L}{R_{\text{eq}}}$$

$$\omega_{LC} = \frac{1}{\sqrt{LC}}$$

*A* = area  
*B* = magnetic field  
*C* = capacitance  
*F* = force  
*I* = current  
*ℓ* = length  
*L* = inductance  
*n* = number of loops per unit length  
*N* = number of loops  
*q* = charge  
*r* = radius, distance, or position  
*R* = resistance  
*t* = time  
*U* = potential energy  
*v* = velocity or speed  
*ε* = emf  
*μ* = magnetic permeability  
*τ* = time constant  
*Φ* = flux  
*ω* = angular frequency



MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\Delta x = \int v_x(t) dt$$

$$\Delta v_x = \int a_x(t) dt$$

$$\bar{x}_{\text{cm}} = \frac{\sum m_i \bar{x}_i}{\sum m_i}$$

$$\bar{r}_{\text{cm}} = \frac{\int \bar{r} dm}{\int dm}$$

$$\lambda = \frac{d}{d\ell} m(\ell)$$

$$\bar{a}_{\text{sys}} = \frac{\sum \bar{F}}{m_{\text{sys}}} = \frac{\bar{F}_{\text{net}}}{m_{\text{sys}}}$$

$$|\bar{F}_g| = G \frac{m_1 m_2}{r^2}$$

$$|\bar{F}_f| \leq |\mu \bar{F}_N|$$

$$\bar{F}_s = -k \Delta \bar{x}$$

$$a_c = \frac{v^2}{r} = r \omega^2$$

$$T = \frac{1}{f}$$

$$K = \frac{1}{2} m v^2$$

$$W = \int_a^b \bar{F} \cdot d\bar{r}$$

$$\Delta K = \sum W_i = \sum F_{\parallel, i} d_i$$

$$\Delta U = - \int_a^b \bar{F}_{\text{cf}}(r) \cdot d\bar{r}$$

$$F_x = - \frac{dU(x)}{dx}$$

$$U_s = \frac{1}{2} k (\Delta x)^2$$

$$U_G = -G \frac{m_1 m_2}{r}$$

$$\Delta U_g = mg \Delta y$$

$a$  = acceleration  
 $E$  = energy  
 $f$  = frequency  
 $F$  = force  
 $h$  = height  
 $J$  = impulse  
 $k$  = spring constant  
 $K$  = kinetic energy  
 $\ell$  = length  
 $m$  = mass  
 $M$  = mass  
 $p$  = momentum  
 $P$  = power  
 $r$  = radius, distance, or position  
 $t$  = time  
 $T$  = period  
 $U$  = potential energy  
 $v$  = velocity or speed  
 $W$  = work  
 $x$  = position or distance  
 $y$  = height  
 $\lambda$  = linear mass density  
 $\mu$  = coefficient of friction

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = \frac{dW}{dt}$$

$$\bar{p} = m \bar{v}$$

$$\bar{F}_{\text{net}} = \frac{d\bar{p}}{dt}$$

$$\bar{J} = \int_{t_1}^{t_2} \bar{F}_{\text{net}}(t) dt = \Delta \bar{p}$$

$$\bar{v}_{\text{cm}} = \frac{\sum \bar{p}_i}{\sum m_i} = \frac{\sum m_i \bar{v}_i}{\sum m_i}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$v = r \omega$$

$$a_T = r \alpha$$

$$\bar{\tau} = \bar{r} \times \bar{F}$$

$$I_{\text{tot}} = \sum I_i = \sum m_i r_i^2$$

$$I = \int r^2 dm$$

$$I' = I_{\text{cm}} + M d^2$$

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$

$$W = \int \tau \cdot d\theta$$

$$\bar{L} = \bar{r} \times \bar{p} = I \bar{\omega}$$

$$\Delta L = \int \tau dt$$

$$\Delta x_{\text{cm}} = r \Delta \theta$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

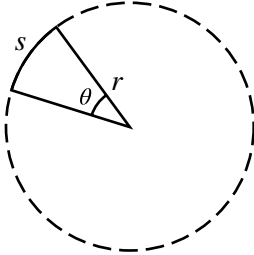
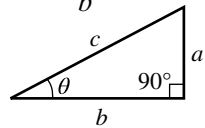
$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$T_{\text{phys}} = 2\pi \sqrt{\frac{I}{mgd}}$$

$$x = x_{\text{max}} \cos(\omega t + \phi)$$

$a$  = acceleration  
 $d$  = distance  
 $f$  = frequency  
 $F$  = force  
 $I$  = rotational inertia  
 $k$  = spring constant  
 $K$  = kinetic energy  
 $\ell$  = length  
 $L$  = angular momentum  
 $m$  = mass  
 $M$  = mass  
 $p$  = momentum  
 $r$  = radius, distance, or position  
 $t$  = time  
 $T$  = period  
 $v$  = velocity or speed  
 $W$  = work  
 $x$  = position or distance  
 $\alpha$  = angular acceleration  
 $\theta$  = angle  
 $\tau$  = torque  
 $\phi$  = phase angle  
 $\omega$  = angular frequency or angular speed

**GEOMETRY AND TRIGONOMETRY**

<p><b>Rectangle</b> <math>A = bh</math></p> <p><b>Triangle</b> <math>A = \frac{1}{2}bh</math></p> <p><b>Circle</b> <math>A = \pi r^2</math> <math>C = 2\pi r</math> <math>s = r\theta</math></p>	<p><b>Rectangular Solid</b> <math>V = \ell wh</math></p> <p><b>Cylinder</b> <math>V = \pi r^2 \ell</math> <math>S = 2\pi r \ell + 2\pi r^2</math></p> <p><b>Sphere</b> <math>V = \frac{4}{3}\pi r^3</math> <math>S = 4\pi r^2</math></p>		<p><math>A = \text{area}</math> <math>b = \text{base}</math> <math>C = \text{circumference}</math> <math>h = \text{height}</math> <math>\ell = \text{length}</math> <math>r = \text{radius}</math> <math>s = \text{arc length}</math> <math>S = \text{surface area}</math> <math>V = \text{volume}</math> <math>w = \text{width}</math> <math>\theta = \text{angle}</math></p>	<p><b>Right Triangle</b> <math>a^2 + b^2 = c^2</math> <math>\sin \theta = \frac{a}{c}</math> <math>\cos \theta = \frac{b}{c}</math> <math>\tan \theta = \frac{a}{b}</math></p> <div style="text-align: right;">  </div>
--	--	---	--	--

VECTORS	CALCULUS	IDENTITIES
$\vec{A} \cdot \vec{B} = AB \cos \theta$ $ \vec{A} \times \vec{B}  = AB \sin \theta$ $\vec{r} = (A\hat{i} + B\hat{j} + C\hat{k})$ $\vec{C} = \vec{A} + \vec{B}$ $\vec{C} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$	$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$ $\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\frac{d}{dx}(\ln ax) = \frac{1}{x}$ $\frac{d}{dx}[\sin(ax)] = a \cos(ax)$ $\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$ $\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$ $\int e^{ax} dx = \frac{1}{a}e^{ax}$ $\int \frac{dx}{x+a} = \ln x+a $ $\int \cos(ax) dx = \frac{1}{a} \sin(ax)$ $\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$	$\log(a \cdot b^x) = \log a + x \log b$ $\sin^2 \theta + \cos^2 \theta = 1$ $\sin(2\theta) = 2 \sin \theta \cos \theta$ $\frac{\sin \theta}{\cos \theta} = \tan \theta$

